



**Faculty of science**  
**Department of Mathematics**

**Final Term Exam (2<sup>nd</sup> Term)**  
**Fourth year student (Math)**  
**Course: Complex Analysis**  
**Code: 412 M**

**Time : 3 Hours**

**Points : 50 Points**

**Date: Tuesday, 21 May 2019**



**Assiut University**

الإمتحان مُكون من (خَمسة أسئلة) – تقع الأسئلة في (صفحتين) – مطلوب الإجابة عنها جميعاً

**1-a) Write  $f(z) = \sin \frac{1}{z}$ ,  $z \in \mathbb{C} \setminus \{0\}$  in the form  $u(x, y) + i v(x, y)$ . (3 Pts)**

**b) Let  $D = \{z = (x, y) \in \mathbb{C} : 0 \leq x \leq \pi, \pi \leq y \leq 2\pi\} \subset \mathbb{C}$ ,  $f: \mathbb{C} \rightarrow \mathbb{C}$ .**

**Give a precise geometric description of the image  $f(D)$  under the**

**transformation  $f(z) = e^z$ . (4 Pts)**

**c) Discuss the existence of the limit  $\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}}\right)^2$ . (3 Pts)**

**2-a) Prove that if  $f(z)$  is uniformly continuous in a domain  $D \subset \mathbb{C}$ , then**

**$f(z)$  is continuous in  $D$ . (3 Pts)**

**b) Give an example of a complex-valued function that is continuous,**

**but not uniformly continuous. (3 Pts)**

**c) Use the polar form of the Cauchy-Riemann equations to verify that**

**$f'(z) = 2z$  when  $f(z) = z^2$ . (4 Pts)**

**3-a) Determine the values of  $z$  for which the function  $f(z) = \frac{1}{\sqrt{3} \sin z - \cos z}$  fails to be analytic. (5 Pts)**

**b) Suppose that  $f(z) = u(x, y) + i v(x, y)$  is analytic in a domain  $D$ .**

**Show that  $u(x, y)$  and  $v(x, y)$  are harmonic in  $D$ . (5 Pts)**

**Please turn the page**

4-a) Show that  $\left(\frac{n+2i}{n+1}\right)$  is a Cauchy sequence. (4 Pts)

b) Let  $(z_n) = \left(\sum_{k=0}^n \left(\left(\frac{3}{5}\right)^k + \left(\frac{4}{5}\right)^k i\right)\right)$ . Does  $(z_n)$  converges. If it does, find  $\lim_{n \rightarrow \infty} z_n$  in the form  $x + iy$ ,  $x, y \in \mathbb{R}$ . (3 Pts)

c) Find the radius of convergence of  $\sum_{k=0}^{\infty} \frac{k^k}{k!} z^k$ . (3 Pts)

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5) In each part , assume that the curve is oriented counterclockwise.

a) Evaluate the following integrals:

i)  $\int_{c_1} \bar{z} dz$ ,  $c_1$  is the triangle  $ABC$ ,  $A = 0$ ,  $B = 1 + i$ ,  $C = -2$ . (3 Pts)

ii)  $\int_{c_2} \frac{e^z + \sin z}{z^2 - 2z - 8} dz$ ,  $c_2$  is the circle  $|z - 1| = 2$ . (3 Pts)

b) Estimate  $\left| \int_c \frac{\log(z^2)}{z^2} dz \right|$ , where  $c$  is the circle  $|z| = 100$ . (4 Pts)

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*Dr. A.M. Gaddeek* ,,, *With best wishes* ,,, *Signature Gaddeek*

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Level 4 Computer Science Section Time: 3 H  
Science Faculty Operation Research 426 R June 2016  
Math. Depart Total degree (50)

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*Answer four questions only from the following questions:*

- 1-a) Derive the necessary and sufficient conditions for  $x^*$  to be a local minimum point for the problem:

$$\text{Min } f(x) \quad (6 \text{ points})$$

- b) Find the extreme points and extreme functions of the following function

$$f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 12 \quad (6.5 \text{ points})$$

- 2-a) The Newton-Raphson method used to solve the non-linear equation  $f(x) = 0$  can be easily modified to find the extreme values of the single variable problems. Derive this modified formula and then use it to solve the following problem

$$\text{Min } f(\underline{x}) = x_1^2 + 2x_2^2 - 4x_1 - 2x_1x_2, \quad \underline{x}^0 = (0.1) \quad , \varepsilon = 10^{-4}$$

for 2 iterations only (7.5 points)

- b) Verify whether the vector  $d = (-3, 10, 12)$  at the point  $(1, 2, 3)$  is a descent direction for the function

$$f(x) = x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3 \quad (5 \text{ points})$$

*Please See the Next Page*



3-a) Derive the necessary and the sufficient conditions for the  
given problem  $\text{Min } f(\underline{x}) , \underline{x} \in R^n ,$

where  $f(\underline{x})$  is a continuous and differentiable function for  
all needed order (6.5 points)

-b) Find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6 \quad (6 \text{ points})$$

4-a) Derive the necessary condition of the constrained  
variation method used to solve the following problem

$$\begin{aligned} &\text{Minimize } f(x_1, x_2) \\ &\text{Subject to } C(x_1, x_2) = 0. \end{aligned} \quad (6.5 \text{ points})$$

-b) Use the derived method to solve the problem

$$\begin{aligned} &\text{Minimize } f(\underline{x}) = (x_1 - 1)^2 + x_2^2 \\ &\text{Subject to } x_1^2 + x_2^2 = 4. \end{aligned} \quad (6 \text{ points})$$

5-a) Use the constraint variation method to derive the Lagrange  
multiplier method for the following problem

$$\text{Mini } f(x_1, x_2) \quad \text{Subject to } g(x_1, x_2) = 0 \quad (6 \text{ points})$$

-b) Use the Lagrange multiplier method to find the extreme  
points of the problem

$$\begin{aligned} &\text{Maximize } f(\underline{x}) = x_1^2 + x_2^2 - 2x_1 + 1 \\ &\text{S.t } g(\underline{x}) = x_1^2 + x_2^2 = 4 \end{aligned} \quad (6.5 \text{ points})$$

With my best wishes

Taha Elgindy



Assiut University	Numerical Analysis (2)	Date: 28/5/2019
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

**Answer 5 (five) questions ONLY from the following (grades equally distributed):**

1. (a) Use the midpoint method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 1, \quad y(0) = 0.5, \quad h = 0.5.$$

- (b) Drive the finite-difference method of order  $h^2$  to approximate the solution of the nonlinear boundary value problem  $y'' = f(x, y, y')$ ,  $y(a) = \alpha$ ,  $y(b) = \beta$ .

2. (a) Suppose  $f$  is continuous and satisfies a Lipschitz condition with constant  $L$  on  $D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$  and that a constant  $M$  exists with  $|y''(t)| \leq M$ , for all  $t \in [a, b]$ , where  $y(t)$  denotes the unique solution to the initial-value problem  $y' = f(t, y)$ ,  $a \leq t \leq b$ ,  $y(a) = \alpha$ .

Let  $w_0, w_1, \dots, w_N$  be the approximations generated by Euler's method for some positive integer  $N$ . Prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], \quad i = 0, 1, \dots, N.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, \quad y(0) = 0, \quad y(1) = 1, \quad h = 0.25.$$

3. (a) Write the second-order initial-value problems  $y'' = p(x)y' + q(x)y + r(x)$ ,  $a \leq x \leq b$ ,  $y(a) = \alpha$ ,  $y'(a) = 0$ , and  $y'' = p(x)y' + q(x)y$ ,  $a \leq x \leq b$ ,  $y(a) = 0$ ,  $y'(a) = 1$ , as first-order systems, and derive the equations necessary to solve the systems using the fourth-order Runge-Kutta method for systems.

- (b) Determine the Court factorization of the symmetric tri-diagonal matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}, \text{ and use this factorization to solve the linear system}$$

$$2x_1 - x_2 = 1, \quad -x_1 + 2x_2 - x_3 = 0, \quad -x_2 + 2x_3 - x_4 = 0, \quad -x_3 + 2x_4 = 1.$$

4. (a) Discuss the stability considerations of the forward-difference method when used to solve the heat equation

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2}, \quad 0 < x < l, \quad t > 0,$$

subject to the conditions

$$u(0, t) = y(l, t) = 0, \quad t > 0, \text{ and } u(x, 0) = f(x), \quad 0 \leq x \leq l.$$

باقى الاسئلة فى الخلف

(b) Use Newton's method with  $\mathbf{x}^{(0)} = \begin{bmatrix} 2.1 \\ -1.3 \end{bmatrix}$  to compute  $\mathbf{x}^{(2)}$  for the following nonlinear system:  $x_1^2 - x_2^2 + 2x_2 = 0$ ,  $2x_1 + x_2^2 - 6 = 0$ .

5. (a) Derive the scheme arising from the finite-difference method to the Poisson equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} + \frac{\partial^2 u(x, t)}{\partial y^2} = f(x, y),$$

on  $R = \{(x, y) | a \leq x \leq b, c \leq y \leq d\}$ , with  $u(x, y) = g(x, y)$  for  $(x, y) \in S$  where  $S$  denotes the boundary of  $R$ .

(b) Draw three diagrams showing the interaction of the nodes for determining an approximation at  $(x_i, t_j)$  to the heat equation by using forward-difference, backward-difference, and Crank-Nicolson methods.

6. Derive the linear systems arising from backward-difference method and Crank-Nicolson method at any point  $(x_i, t_j)$  to the heat equation

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2}, \quad 0 < x < l, \quad t > 0,$$

Subject to the conditions

$$u(0, t) = u(l, t) = 0, \quad t > 0, \text{ and } u(x, 0) = f(x), \quad 0 \leq x \leq l.$$

انتهت الاسئلة

د. شعبان علي بكر



2018/2019  
2<sup>nd</sup> Term  
Date: May, 29, 2019

Final Exam for Level 4  
Subject: Distributed Computation, MC452

Time: 2 Hours  
50 marks

Mathematics Dept.  
Faculty of Science  
Assiut University

Answer the following questions (50 marks)

<p><b>Q. 1.</b></p> <p><b>Complete TEN of the following:</b></p> <ul style="list-style-type: none"> <li>• .....(a).... Programming in a language that supports concurrency explicitly</li> <li>• .....(b)..... One instruction stream is broadcast to all processors</li> <li>• In Multicomputers, Processors are connected by.....(c)..... and Data is passed between processors using .....(d).....</li> <li>• Ratio of switch nodes to processor nodes is 1:1 called .....(e).....</li> <li>• Directory-based Protocol Sharing Status are ...(f)... , .....(g)..... and .....(h).....</li> <li>• a parfor loop cannot contain .....(i)..... , .....(j)..... and .....(k).....</li> </ul>	<p>( 10 marks)</p>		
<p><b>Q. 2.</b></p> <p>a) After running this code, what are the type and the value of each variable? (6 marks)</p> <table border="1" data-bbox="191 1400 1276 1697"> <tr> <td data-bbox="191 1400 730 1697"> <p><b>i)</b>    <code>x = 2;</code>            <code>parfor i=1:100</code>            <code>  A(i)=i*x;</code>            <code>  S=S+A(i);</code>            <code>end</code></p> </td><td data-bbox="730 1400 1276 1697"> <p><b>ii)</b>    <code>spmd(4)</code>            <code>if labindex==1</code>            <code>  a=rand(2,2)</code>            <code>else</code>            <code>  a=rand(1,1)</code>            <code>end</code>            <code>end</code></p> </td></tr> </table> <p>b) What are the main characteristics of multiprocessors (7 marks)</p>	<p><b>i)</b>    <code>x = 2;</code>            <code>parfor i=1:100</code>            <code>  A(i)=i*x;</code>            <code>  S=S+A(i);</code>            <code>end</code></p>	<p><b>ii)</b>    <code>spmd(4)</code>            <code>if labindex==1</code>            <code>  a=rand(2,2)</code>            <code>else</code>            <code>  a=rand(1,1)</code>            <code>end</code>            <code>end</code></p>	<p>(13 marks)</p>
<p><b>i)</b>    <code>x = 2;</code>            <code>parfor i=1:100</code>            <code>  A(i)=i*x;</code>            <code>  S=S+A(i);</code>            <code>end</code></p>	<p><b>ii)</b>    <code>spmd(4)</code>            <code>if labindex==1</code>            <code>  a=rand(2,2)</code>            <code>else</code>            <code>  a=rand(1,1)</code>            <code>end</code>            <code>end</code></p>		



<p><b>Q. 3.</b></p> <p>a) Find the errors in the following code and correct them if found (6marks)</p> <table border="1" data-bbox="183 365 1256 705"> <tr> <td data-bbox="183 365 657 705"> <pre>i) k=5 parfor idx = 0:0.2:1; x(idx) = x(idx-1) + k; end</pre> </td> <td data-bbox="657 365 1256 705"> <pre>ii) function bb data = rand(5,5); means = zeros(1,5); parfor X = 1:5 y.mean = mean(data(:,X)); means(X) = y.mean; end disp(means);</pre> </td> </tr> </table> <p>b) Use the fact that <math>\pi = \int_0^1 \frac{4}{1+x^2} dx</math> to approximate pi in pmode. (6 marks)</p>	<pre>i) k=5 parfor idx = 0:0.2:1; x(idx) = x(idx-1) + k; end</pre>	<pre>ii) function bb data = rand(5,5); means = zeros(1,5); parfor X = 1:5 y.mean = mean(data(:,X)); means(X) = y.mean; end disp(means);</pre>	<p>(12 marks)</p>
<pre>i) k=5 parfor idx = 0:0.2:1; x(idx) = x(idx-1) + k; end</pre>	<pre>ii) function bb data = rand(5,5); means = zeros(1,5); parfor X = 1:5 y.mean = mean(data(:,X)); means(X) = y.mean; end disp(means);</pre>		
<p><b>Q. 4.</b></p> <p>a) What are the differences between (12 marks)</p> <ol style="list-style-type: none"> <li>Shared and Switched Media Interconnection Networks</li> <li>Extend compilers and Extend languages</li> <li>Linear Network and shuffle-exchange Networks</li> </ol> <p>b) Suppose that matrix A represents test scores, the rows of which denote different classes. You want to calculate the difference between the average score and individual scores for each class. Rewrite this code to execute faster (3 marks)</p> <pre>A = [97 89 84; 95 82 92; 64 80 99; 76 77 67; ... 88 59 74; 78 66 87; 55 93 85]; mA = mean(A); B = zeros(size(A)); for n = 1:size(A,2) B(:,n) = A(:,n) - mA(n); end</pre>	<p>(15 marks)</p>		

Best Wishes, Dr. Hanaa A. Sayed

**Answer the following questions: (50 Marks)**

**Question 1: Answer the following questions: (10 Marks)**

- 1-What is NIST definition for Computer Security?
- 2-Explain the term CIA Triad?
- 3- Mention some of the computer security challenges?

**Question 2: Answer the following questions: (10 Marks)**

- 1- Based on RFC 2828, mention the four kinds of threat consequences and list the kinds of attacks that result in each consequence?
- 2- Compare between passive and active attacks?
- 3- Write the X.800 and RFC 2828 definitions for security service?

**Question 3: Answer the following questions: (10 Marks)**

- 1-Explain the symmetric encryption ingredients?
- 2-Explain the methods used to attack symmetric encryption?
- 3-Explain DES?

**Question 4: Answer the following questions: (10 Marks)**

- 1- What are the general means of authenticating a user's identity?
- 2-What is NIST definition to the term malware?
- 3-Malware are mainly classified into two broadly categories. What are these categories? Discuss another method for malware classification?

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**Question 5: Answer the following questions: (10 Marks)**

- 1-What is NIST definition for DoS attack?
  - 2- Explain poison packet?
  - 3- Explain cyberslam attack?
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Answer the following questions:

(50 Marks)

**Question 1:**

(12 Marks)

- What is data mining?
- What are data mining functionalities?
- Explain the differences between Knowledge-discovery database and data mining.

**Question 2:**

(10 Marks)

- What are the classification, clustering and association rule processes?
- Explain the major tasks in data preprocessing.

**Question 3:**

(13 Marks)

- Briefly outline the data transformation methods
- Use the three methods below to normalize the following group of data:  
200; 300; 400; 600; 1000.
  - min-max normalization by setting min = 0 and max = 1.
  - z-score normalization.
  - decimal scaling normalization.

**Question 4:**

(15 Marks)

The following table consists of training data. Let  $X = (\text{age} \leq 30, \text{Income} = \text{medium}, \text{Student} = \text{yes}, \text{Credit\_rating} = \text{Fair})$ . What would a naive Bayesian classification of the status  $X$ ?

age	income	student	credit_rating	com
$\leq 30$	high	no	fair	no
$\leq 30$	high	no	excellent	no
31...40	high	no	fair	yes
$> 40$	medium	no	fair	yes
$> 40$	low	yes	fair	yes
$> 40$	low	yes	excellent	no
31...40	low	yes	excellent	yes
$\leq 30$	medium	no	fair	no
$\leq 30$	low	yes	fair	yes
$> 40$	medium	yes	fair	yes
$\leq 30$	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
$> 40$	medium	no	excellent	no

Best Wishes

Dr. Mohamed Mostafa Darwish





امتحان الفصل الثاني ٢٠١٨ / ٢٠١٩ م الزمن: ثلاث ساعات  
كلية العلوم قسم الرياضيات

المادة: نموذج رياضية (٤٣٤ر) التاريخ: ١٥-٦-٢٠١٩  
للفرقة الرابعة علوم شعبة الرياضيات

أجب عمايلي: (العظمى خمسون والسؤال عليه عشر درجات)

١- ناقش مسألة رايلى- تايلور لاستقرار مائع فوق آخر في وجود الجاذبية الأرضية وبين متى يكون النظام مستقر من عدمه وأهمية هذه المسألة من وجهة نظر الطاقة .

٢- أوجد الحل التقريبي للنظام الفيزيائي التذبذبي  $\ddot{q} + \omega^2 q = \varepsilon \dot{q}$  عندما يكون التردد الطبيعي للنظام يعتمد علي الزمن .

٣- بعيدا عن الطريقة البيانية أوجد أفضل خط مستقيم بطريقتين بحيث يمثل النتائج الآتية ما أمكن وبين أيهما أدق .

x	2	3	4	5	6
y	1.1	1.9	2.6	3.4	4.3

- ٤- ١- عرف النمذجة الرياضية وأذكر فقط أساسياتها الخمسة . (درجتان)
- ٢- تقسم النماذج الرياضية الي ثلاث مجموعات أساسية ناقشها بإيجاز دون ذكر أمثلة . (درجتان)
- ٣- بين متى نلجأ الي النموذج الفيزيائي والوسائل الثلاث لتطويره والفائدة من ذلك . (درجتان)
- ٤- ناقص الغرض من دراسة موضوع النمذجة الرياضية في أربع نقاط أساسية . (درجتان)
- ٥- ماذا تعلمت من موضوع النمذجة الرياضية (درجتان)

٥- استخدم النمذجة الرياضية في الحصول علي:

- ١- المعادلة الموجية في بعد واحد من خلال وتر مهتز (٥درجات)
- ٢- التوتر السطحي لفقااعة الصابون (٥درجات)

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أعده: أ.د. محمود حامد عبيد الله راجعه: أ.د. جمال مختار محمود

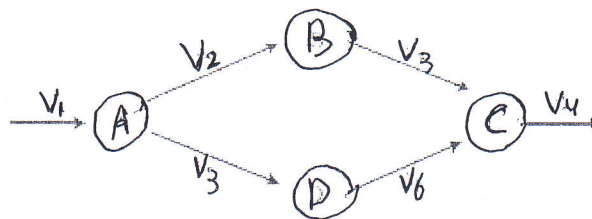
Answer the following questions (12.5 Degree for each question):

**Q1- Answer only Four points.**

- 1- Draw a Venn diagram that explains the domain of systems biology.
- 2- Draw the *in silico* cell.
- 3- Show the cell purpose. Formulate this purpose as a mathematical function.
- 4- Write the relation between flux and the enzyme abundance.
- 5- Explain how the proteomics data can be used to reduce the flux spaces.

**Q2-**

A- For the next network, find **manually** the null space, where  $v_4$  and  $v_6$  are the free variables.



B- Answer only two points.

- 1- Compare between the linear and the convex spaces.
- 2- What are the main members in the model object in the Matlab?
- 3- Discuss briefly the aim of SBML format.

**Q3- Answer only Four points.**

- 1- The  $k_{cat}$  for yeast glucose transporter is 200 per second. Is it possible yeast consumes glucose with the rate of 10 mmol/gDW/h. Write the reason for your resulting answer.
- 2- Write the mathematical formulation of ME-model.
- 3- Compare between FBA and ME-model.
- 4- Explain the required steps for running FBA simulations.
- 5- Write the Matlab code that simulates the gene deletion with FBA.

**Q4- Answer only Four points.**

- 1- Explain the Excel file that contains the genome-scale metabolic model.
- 2- Draw a figure to explain the main steps for generating the draft model.
- 3- Explain the meaning of this sentence: "We can leave the gap in the genome-scale metabolic model".
- 4- Write the relation between the genes and reactions.
- 5- Give an example of a reaction that the metabolites can be written as charged or neutral formulas.



المادة : معادلات تفاضليه جزئية

امتحان الفصل الدراسي الثاني للعام الدراسي 2018 – 2019

الزمن : ثلاث ساعات

المستوي الرابع -مقرر 414 ر

كلية العلوم

الدرجة الكلية : 50 درجة

التاريخ 2019 / 6 / 19

قسم الرياضيات

اجب عن خمسة فقط من الاسئلة الاتية :- ( 10 درجات لكل سؤال: 5 درجات لكل فقره)

(1) اوجد الحل الكامل و الحل المفرد- ان وجد – لكل من المعادلتين الاتيتين

(i)  $(1 - x^2)yp^2 + x^2q = 0$

(ii)  $z = px + qy - 2\sqrt{pq}$

(2) اوجد الحل الكامل للمعادلة الاتية :-  $p^2 = z^2(1 - pq)$

(i) باستخدام طريقة شاربت

(ii) باستخدام طريقة جاكوبي

(3) (i) اثبت ان النظام الاتي متوافق واوجد حله :-  $xp = yq$  ,  $z(xp + yq) = 2xy$

(ii) حل المعادلة الاتية :-  $t - 2xq + x^2z = (x - 2)e^{3x+2y}$

(4) استخدم تعويضا مناسباً لإيجاد حل كل من المعادلتين الاتيتين :-

(i)  $pq = x^m y^n z^{2l}$

(ii)  $(x^2 D_1^2 + 2xy D_1 D_2 + y^2 D_2^2)z = x^m y^n$

(5) بين الصورة القياسية التي عليها كل من المعادلتين الاتيتين ثم اوجد حل كل منهما :-

(i)  $2xr - ys + 2p = 4xy^2$

(ii)  $xs + q - xp - z = (1 - y)(1 + \log x)$

(6) استخدم طريقة لابلاس لحل كل من المعادلتين الاتيتين :-

(i)  $x^2r - y^2t + px - qy = x^2$

(ii)  $t - s + p\left(1 + \frac{1}{x}\right) + \frac{z}{x} = 0$

انتهت الأسئلة مع اطيب الامنيات بالنجاح

لجنة الممتحنين د. مجدي كامل الجندي ، د. محمد عبد الله عبد الرازق





الزمن : 3 ساعات	إمتحان نهائي الفصل الثاني - إحصاء رياضي	
الدرجة: 50	رابعة علوم شعبة رياضيات	قسم الرياضيات - كلية العلوم 1440 هـ - 2018 م

أجب عن خمسة فقط من الأسئلة الآتية (10 درجات لكل سؤال): (الأسئلة في صفحتين)

(1) (أ) إذا أعطيت دالة الكتلة الاحتمالية للمتغيرين  $X$  و  $Y$  بالجدول:

x \ y	-1	-0.5	0.5	1
-2	1/8	0	0	0
-1	0	1/4	0	0
1	0	0	1/2	0
2	0	0	0	1/8

فأوجد: (i)  $p_X(x)$ ,  $p_Y(y)$  (ii)  $p_{X|Y}(x|2)$ ,  $V(X|Y=2)$  (iii)  $\rho(X, Y)$

(ب) المتغير العشوائي  $X$  له الكثافة الاحتمالية:

$$f_X(x) = 4x e^{-2x}, x > 0 \quad (= 0, \text{o.w})$$

$$Z = \begin{cases} -1, & 0 < x < 2 \\ -2, & 2 \leq x < 4 \\ 1, & 4 \leq x < 6 \\ 2, & x \geq 6 \end{cases} \quad \text{وإذا كان:}$$

فأوجد التوزيع الاحتمالي للمتغير العشوائي  $Z$ .

(2) (أ) إذا كان المتغيران  $X$  و  $Y$  مستقلان ويتبعان توزيع بواسون بالبارامترين  $\lambda_1$  و  $\lambda_2$

على الترتيب. أوجد توزيع المتغير  $Z$  حيث:  $Z = X + Y$ .

(ب) عينة عشوائية  $X_1, X_2, \dots, X_n$  حجمها  $n$  مسحوبة من مجتمع يتبع توزيع ذو الحدين بالمعلمتين  $m, p$ ، أوجد مقدار الإمكان الأكبر للبارامتر  $p$  ثم أدرس خواص التحيز والإتساق و عدم التحيز بأقل تباين (MVUE).

(3) (أ) إذا كان المتغير العشوائي  $X$  يخضع للتوزيع المعتدل القياسي، أوجد توزيع

المتغير العشوائي  $Z$  حيث  $Z = X^2$  بطريقتي دالة التوزيع والتحويل.

(ب) إذا كان المتغير العشوائي  $X$  يمثل الزمن الذي يأخذه عامل فني في تنفيذ عمل ما مكلف به، وكانت دالة الكثافة لهذا المتغير هي:

$$f_X(x) = e^{-(x-\theta)}, x > \theta, (\theta > 0) \quad (= 0, \text{o.w})$$

وبفرض أن  $X_1, X_2, \dots, X_n$  عينة عشوائية وكانت  $X_{(1)} = \min\{X_1, \dots, X_n\}$  و

(i) دالة الكثافة الاحتمالية للمتغير  $X_{(1)}$  فأوجد:  $X_{(n)} = \max\{X_1, \dots, X_n\}$

(ii) دالة الكثافة الاحتمالية للمتغير  $X_{(n)}$

(4) (أ) إذا كانت  $X_1, X_2, \dots, X_n$  عينة حجمها  $n$  مسحوبة من مجتمع يتبع التوزيع التالي:  $f(x; \theta) = \theta x^{\theta-1}, 0 < x < 1, (\theta > 0) (= 0, o.w)$  أوجد مقدر الإمكان الأكبر للمعلمة  $\theta$ .  
(ب) إذا أعطيت دالة الكثافة الاحتمالية المفصلية:

$$f_{X,Y}(x,y) = x e^{-x(y+1)}, x > 0, y > 0. (= 0, o.w)$$

فاوجد: (i)  $E(X^r | Y = y)$  (ii)  $V(X | Y = y)$  (iii)  $F_{X,Y}(x, y)$

(5) (أ) بفرض أن مجتمعاً معتدلاً متوسطه  $\mu$  وتباينه  $\sigma^2$  معلوم القيمة، أخذت عينة من هذا المجتمع حجمها  $n$  وكان متوسطها  $\bar{x}$ ، أثبت أن  $100\%(1 - \alpha)$  فترة ثقة لمتوسط المجتمع  $\mu$  (حيث  $0 < \alpha < 1$ ) تكتب على الصورة:

$$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

(ب) (a) بفرض أن لدينا عينتين مستقلتين حجوما  $n_1$  و  $n_2$  مسحوبتان من مجتمعين

$$S_i^2 = \frac{\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2}{n_i - 1}, i = 1, 2 \text{ وبفرض أن } \sigma^2, \text{ ومعتدلين لهما نفس التباين } \sigma^2,$$

(i) أثبت أن  $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$  مقدر غير متحيز ل  $\sigma^2$  (ii) أوجد  $V(S_p^2)$

(b) يقوم مصنع ما بتجميع نوع من أجهزة الكمبيوتر، قررت إدارته التعاقد مع فنيين جدد بعد خضوعهم لفترة من التدريب لكي يصلوا إلى أقصى قدر من الكفاءة، وأقترحت الإدارة الفنية أسلوباً جديداً للتدريب. وأجريت دراسة للمقارنة بين الأسلوب المقترح والأسلوب التقليدي المستخدم من قبل، فأختيرت عينتان حجم كل منهما 9 موظفين جدد، وسجلت بالدقائق الفترات الزمنية التي يأخذها كل منهم في التجميع، فكانت متوسطات العينتين هما  $\bar{x}_1 = 31.56$ ،  $\bar{x}_2 = 35.22$  وتبايناتها  $S_1^2 = 20.028$ ،  $S_2^2 = 24.445$  أوجد  $95\%$  فترة ثقة للفرق بين المتوسطين  $(\mu_1 - \mu_2)$  وذلك بفرض أن أزمان التجميع في المجتمعين تتبع توزيعات معتدلة مجهولة ومتساوية التباينات. (أستخدم القيمة الجدولية:  $t_{0.025, 16} = 2.12$ ).

(6) (أ) إذا كان المتغيران  $X, Y$  مستقلين بحيث أن  $X \sim \chi^2(v_1)$  و  $Y \sim \chi^2(v_2)$  وكان  $F = \frac{X/v_1}{Y/v_2}$  فاثبت أن:  $F \sim F(v_1, v_2)$ .

(ب)  $X_1, \dots, X_n$  هي عينة عشوائية مسحوبة من مجتمع يتبع دالة الكثافة:

$$f_X(x; \theta) = \frac{1}{\theta + 1} e^{-\frac{x}{\theta+1}}, x > 0, (\theta > -1) (= 0, o.w)$$

(i) أدرس خاصية التحيز لـ  $\bar{X}$  كمقدر لـ  $\theta$ ، وفي حالة تحيزه أستخدمه لإيجاد مقدر غير متحيز لـ  $\theta$ . (ii) أوجد  $\hat{\theta}_M, \hat{\theta}_{ML}$ .

صيغ معاونة:

$$(1) X \sim \text{bin}(m, p) \Rightarrow p_X(x, p) = \binom{m}{x} p^x (1-p)^{m-x}, x = 0, 1, \dots, m$$

$$(2) X \sim \text{Poisson}(\lambda) \Rightarrow p_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, \dots (\lambda > 0)$$

$$(3) X \sim N(0, 1) \Rightarrow f_X = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$$

$$(4) Y \sim \chi^2(k) \Rightarrow f_Y(y; k) = \frac{1}{\Gamma(k/2) 2^{k/2}} y^{(k/2)-1} e^{-y/2}, y > 0$$

$$(5) Z \sim F(v_1, v_2) \Rightarrow f_Z(z) = \frac{\Gamma\left(\frac{v_1+v_2}{2}\right) \left(\frac{v_1}{v_2}\right)^{\frac{v_1}{2}} z^{\frac{v_1}{2}-1} [1 + \left(\frac{v_1}{v_2}\right) z]^{-(v_1+v_2)/2}, z > 0$$

انتهت الأسئلة ،،، موفقين أ.د. عبد الباسط عبد الله أحمد





**Answer the following questions:**

**(50 Marks)**

**I. Complete the following statements from the given table.**

**(20 Marks)**

**Note:** each answer may be used several times.

1	$O(n)$	6	Lexeme	11	Ambiguity	16	Parse tree
2	DFA	7	Priority	12	CFG	17	Two
3	Go to	8	Maximal munch	13	DFS	18	$O(mn^2)$
4	Semantic analysis	9	LR(1)	14	LALR(1)	19	Optimization
5	IR Optimization	10	BFS	15	Tokens	20	Four

a. .... identify the meaning of the overall structure.	( )
b. Any LR(0) grammar is .....	( )
c. .... is the piece of the original program from which we made the token.	( )
d. There are ..... main kinds of finite automata.	( )
e. Every SLR(1) grammar is .....	( )
f. Any regular expression of length $n$ can be converted into an NFA with ..... states. .	( )
g. .... is a formalism for defining languages.	( )
h. Leftmost ..... works on all grammars.	( )
i. Formally, a CFG is a collection of ..... objects. .	( )
j. .... is impractical because its contextual information makes the automaton too big.	( )
k. .... a property of grammars, not languages.	( )
l. .... improve the resulting structure.	( )
m. The worst-case memory usage of ..... is linear.	( )
n. The ..... table maps state/symbol pairs to a next state.	( )
o. High-memory ..... has lower scan time.	( )
p. .... encodes what productions are used, not the order in which those productions are applied.	( )
q. .... system means to pick the rule that was defined first.	( )
r. .... simplify the intended structure.	( )
s. After lexical analysis, we have a series of.....	( )
t. .... match the longest possible prefix of the remaining text.	( )



**II. Put (✓) OR correct the underline text ONLY if it is not correct in the brackets: ( 10 marks)**

- [       ] 1. Sometimes we will discard a lexeme rather than storing it for later use.
- [       ] 2. In LR(1) parsing algorithm, If action [state, t] is reduce  $A \rightarrow \omega$  then shift the input and set state = goto [state, t].
- [       ] 3. Any LL(1) grammar is LR (1).
- [       ] 4. Two LR(1) items have the same reduction if they are identical except for lookahead.
- [       ] 5. Leftmost BFS works on grammars without left recursion.
- [       ] 6. All LR (1) grammars are LALR (1).
- [       ] 7. Some tokens might be associated with lots of different lexemes.
- [       ] 8. SLR (1) is weak because it has no contextual information.
- [       ] 9. A shift/reduce conflict is an error where a shift/reduce parser cannot tell which of many reductions to perform.
- [       ] 10. LR (0) only accepts languages where the handle can be found with no left context.

**III. (a) Consider the following ambiguous grammar:**

**( 4 marks)**

**E   E + E**

**E   E \* E**

**E   int**

**E   (E)**

Explain why this grammar cannot be parsed with an LL(1) parser, even if the parser knew the relative precedence and associativity of addition and multiplication.

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III. (b) Consider the following grammar:

( 6 marks)

$E \rightarrow E + F$

$F \rightarrow F * T$

$F \rightarrow T$

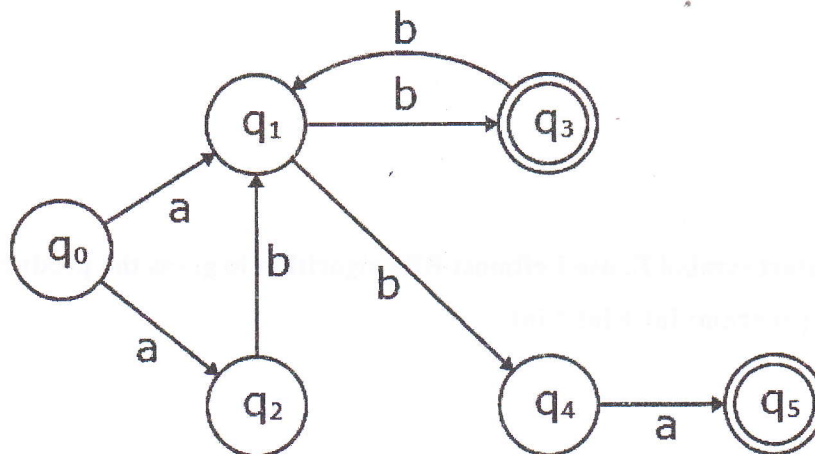
$T \rightarrow \text{int}$

$T \rightarrow (E)$

Beginning with the start symbol  $E$ , use Leftmost BFS algorithm to guess the productions to apply to end up at the user's program:  $\text{int} + \text{int} * \text{int}$

IV. (a) Convert the following NFA to DFA:

(5 Marks)







**Answer the following questions**

**(50 Marks)**

**Question 1:**

**(20 Marks)**

**Complete the following statements from the given table.**

**Note:** each answer may be used several times.

1	Statistical Tests	6	XOR	11	Block	16	PRG
2	Uniform random variable	7	Semantically secure	12	Predictable	17	PRF
3	RC4	8	Salsa20/12	13	Encryption	18	Vigener
4	Non-negligible	9	Caesar	14	Independent	19	CSS
5	Decryption	10	Perfect secrecy	15	S-box	20	Plaintext

- ( ) a. .... Cipher has no key.
- ( ) b. We write  $r \xleftarrow{R} U$  to denote a ..... over  $U$ .
- ( ) c. .... of two strings in  $\{0,1\}^n$  is their bit-wise addition mod 2.
- ( ) d. .... is often randomized.
- ( ) e. A cipher has ..... if  $\Pr[E(k, m_0) = c] = \Pr[E(k, m_1) = c]$ .
- ( ) f.  $E$  is ..... if for all efficient  $A$   $\text{Adv}_{\text{SS}}[A, E]$  is negligible.
- ( ) g. A PRP is a ..... where  $X=Y$  and is efficiently invertible.
- ( ) h. .... must be unpredictable.
- ( ) i. .... Ciphers built by Iteration
- ( ) j. ....  $\Rightarrow \text{key-len} \geq \text{msg-len}$ .
- ( ) k. ....: function  $\{0,1\}^6 \rightarrow \{0,1\}^4$ , implemented as look-up table.
- ( ) l. An algorithm  $A$  is ..... if  $A(x)$  outputs "0" or "1".
- ( ) m. .... is an algorithm that transform ciphertext to plaintext.
- ( ) n. .... used in DVD encryption.
- ( ) o. OTP used ..... function
- ( ) p.  $f: K \times \{0,1\}^n \rightarrow \{0,1\}^n$  a secure .....  
 $\Rightarrow$  3-round Feistel  $F: K^3 \times \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$  a secure PRP
- ( ) q. .... used in HTTPS and WEP.
- ( ) r. .... becomes "more secure" as  $\lambda$  increases.
- ( ) s. Stream ciphers are .....
- ( ) t. .... is always deterministic.

Question 2:

(20 Marks)

Prove each of the following :

1. Y a rand. var. over  $\{0,1\}^n$ , X an indep. uniform var. on  $\{0,1\}^n$  Then  $Z := Y \oplus X$  is uniform var. on  $\{0,1\}^n$ .

2. Two time pad is insecure

3. One Time Pad has perfect secrecy.

4. For all  $f_1, \dots, f_d: \{0,1\}^n \rightarrow \{0,1\}^n$  Feistel network  $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$  is invertible



**Question 3:**

**(10 Marks)**

1. Explain in details the Data Encryption Standard (DES) algorithm with labeled diagrams.

2. Discuss the following :

a) Statistical Tests

b) The term “Advantage” in PRG security

c) Secure PRG

== Best Wishes ==  
*Dr Dafia Nashat*