

Assiut University
Faculty of Science
Mathematics Dept.

Final Exam Spring 17-18
Course: Cryptography
Course code: MC466

Date: 9-5-2018
Level: Fourth
Time: 2 Hour



Answer all questions 10 pt. for each:

Question 1:

1. In points, give a comparison of Block Cipher and Stream Cipher.
2. Using RSA algorithm, perform encryption and decryption for the message="10".

Question 2:

1. In points, briefly describe the steps in the procedure used to construct a knapsack cryptosystem.
2. Using (251,57,120,30,14,7,3,2), perform knapsack cryptosystem to encrypt and decrypt the message="150".

Question 3:

1. In details, define the cryptographic objectives and Shannon's concepts.
2. Use a Hill cipher to encrypt and decrypt the message "hill cipher". Use the following key

$$\begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix}$$

Question 4:

1. In points, give a comparison of DES and AES.
2. Construct a Playfair matrix with the key "occurrence". Encrypt this message: Must see you over Cadogan West

Question 5:

Using Modulus operation, encrypt the message "this is an exercise" using the following ciphers. Also decrypt the ciphertext to get plaintext.

- (i) Caesar cipher with key = 5
- (ii) Affine cipher $y = E(x) = 7x + 3$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Best Wishes

Dr. Mohamed Mostafa Darwish

Assiut University
Faculty of Science
Math. Dep.

4th Mathematics
Complex Analysis 412M
Instructor: Professor Dr. Mohamed Abd El-Rahman Ahmed

Time: 3 hours
Jun : 2013

Answer only five of the following questions:

1- a) **Mention with details** five properties that are satisfied for real functions and not satisfied for corresponding complex functions. (5 degrees)

b) **Show that** $z\bar{z} = |z|^2 \quad \forall z \in C$ and deduce that

$$||z_1| - |z_2|| \leq |z_1 - z_2| \quad \forall z_1, z_2 \in C. \quad (5 \text{ degrees})$$

2- a) **State and prove** Cauchy Integral Formula. (3 degrees)

b) **Solve** the equations $e^{2z-1} = 1$, $\cos(z) = 2$, $\ln(z) = \frac{\pi}{2}i$, $\sinh(z) = i$. (7 degrees)

3- a) **Give and prove** the characterization of the continuity of complex-valued functions. (5 degrees)

b) **Write** Cauchy-Riemann equations at a point. Use these equations to exhibit that the function $f(z) = |z|^2$ hasn't derivative at any point $z \neq 0$. (5 degrees)

4- a) **State and prove** Residue Theorem. (3 degrees)

b) **Show that** $x \cos(x) \cosh(y) + y \sin(x) \sinh(y) - 3e^{-x} \cos(y)$ is a harmonic function.

Also, determine the corresponding analytic function f where the given

function is an imaginary component of f with $f(0) = -3$. Finally, verify that

$$f(z) - f(-z) + 6 \cosh(z) = 0. \quad (7 \text{ degrees})$$

5- a) **Verify that:**

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{|z_2|^2}; \quad z_2 \neq 0, \quad \ln(z) = \ln|z| + i(\arg(z) + 2n\pi); \quad n = 0, \pm 1, \quad (6 \text{ degrees})$$

$$\cosh(z) = \cosh(\operatorname{Re}(z)) \cos(\operatorname{Im}(z)) + i \sinh(\operatorname{Re}(z)) \sin(\operatorname{Im}(z))$$

b) **Using Cauchy Integral Formula for Derivatives of Analytic Functions, find:**

$$\int_C \frac{5z-2}{z^3(z-3)} dz \text{ where } C \text{ is the circle } |z|=2, \text{ described counterclockwise. (4 degrees)}$$

6- a) **State and deduce** the characterization of the limit of complex sequences. (3 degrees)

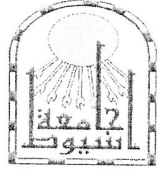
b) (I) **Using Residue Theorem, evaluate** $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ taken

counterclockwise around the circle $|z-2|=2$.

(II) **Write** Laurent series of the function $\frac{z-1}{z^2}$ for the domain $|z-1|>1$. (7 degrees)

The End

GOOD LUCK



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
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ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

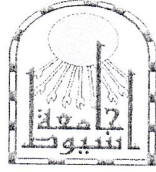
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

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راجع أ. د. جمال مختار محمود

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5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

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انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

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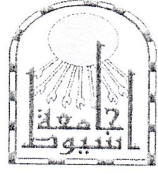
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راجع أ. د. جمال مختار محمود

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Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

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انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

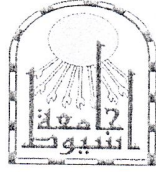
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
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ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$)

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b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

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$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

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$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

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Please See Next Page

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انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

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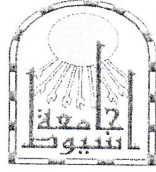
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امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

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عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

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(6 points)

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(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

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b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

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$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

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5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

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Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

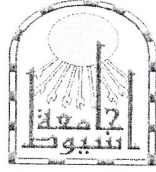
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2, 4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

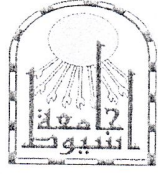
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

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- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $i(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
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Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

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(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

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(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

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$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

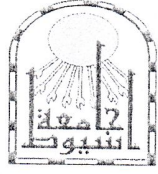
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
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المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

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والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

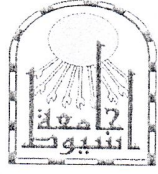
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

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٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

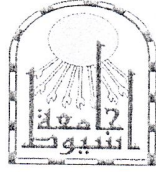
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $i(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

1. The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
2. Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
3. Which of the following is not a reserved word?
a) classes b) void c) static d) public
4. If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
5. Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
6. ----- is the Java assignment operator.
a) == b) := c) = d) =:
7. `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
8. Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
9. What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
10. Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
11. Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
12. When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
13. Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
14. What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
15. The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
16. Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
17. The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" data-bbox="167 1400 1220 1691"> <tr> <td data-bbox="167 1400 694 1691"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td data-bbox="694 1400 1220 1691"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$)

(6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

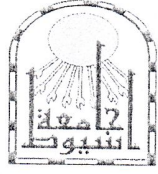
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة (2,4) أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $i(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix.

```
public static boolean isMarkovMatrix(double[][] m)
```

Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.

- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.

- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
II. Java is Object-Oriented and Java is Distributed
III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

1. The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
2. Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
3. Which of the following is not a reserved word?
a) classes b) void c) static d) public
4. If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
5. Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
6. ----- is the Java assignment operator.
a) == b) := c) = d) =:
7. `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
8. Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
9. What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
10. Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
11. Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
12. When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
13. Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
14. What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
15. The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
16. Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
17. The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" data-bbox="167 1400 1220 1691"> <tr> <td data-bbox="167 1400 694 1691"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td data-bbox="694 1400 1220 1691"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$)

(6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

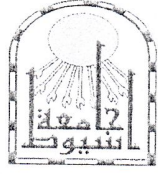
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2, 4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b) grammar	c) regular expression	d) Kleene closure
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow * \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow * A \alpha$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix.

```
public static boolean isMarkovMatrix(double[][] m)
```

Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.

- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.

- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
II. Java is Object-Oriented and Java is Distributed
III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" data-bbox="167 1400 1220 1691"> <tr> <td data-bbox="167 1400 694 1691"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td data-bbox="694 1400 1220 1691"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

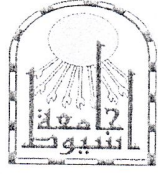
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة (2,4) أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Question2.

(10 Marks)

I. Let G be the grammar:

$S \rightarrow AB$

$A \rightarrow xB \mid \lambda$

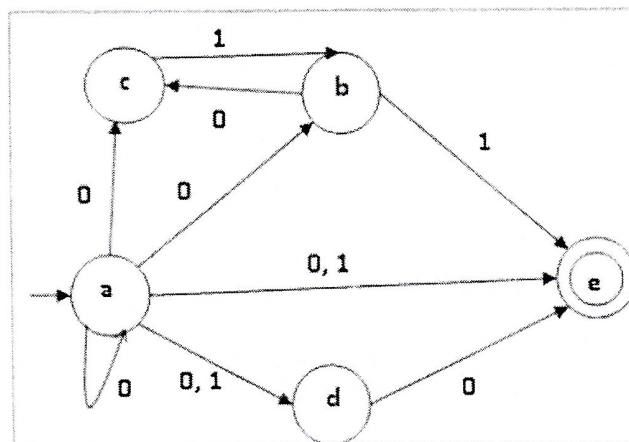
$B \rightarrow yA \mid zB$

1) What are the terminals and non-terminals of this grammar?

3) How might we generate the string $xyyA$?

4) Draw the parse tree for the previous partial derivation

II. Convert the following NFA to DFA.



Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b)	c) grammar	d) regular expression
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow^* \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow^* A\omega$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix.

```
public static boolean isMarkovMatrix(double[][] m)
```

Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.

- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.

- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
II. Java is Object-Oriented and Java is Distributed
III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$)

(6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

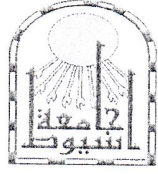
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Question3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question2.

(10 Marks)

I. Let G be the grammar:

$S \rightarrow AB$

$A \rightarrow xB \mid \lambda$

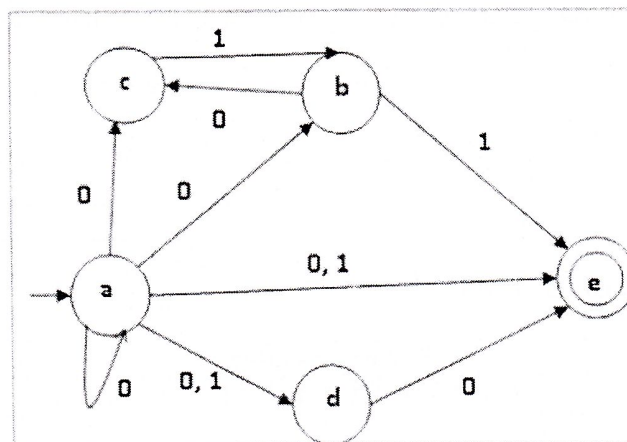
$B \rightarrow yA \mid zB$

1) What are the terminals and non-terminals of this grammar?

3) How might we generate the string $xyyA$?

4) Draw the parse tree for the previous partial derivation

II. Convert the following NFA to DFA.



Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b)	c) grammar	d) regular expression
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow * \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow * A \omega$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix.

```
public static boolean isMarkovMatrix(double[][] m)
```

Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.

- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.

- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
II. Java is Object-Oriented and Java is Distributed
III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

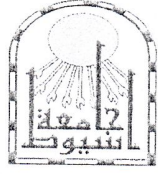
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2, 4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $i(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Question 4. Compare between the following:

(10 Marks)

1) NFA & DFA

2) Leftmost BFS/DFS

Question3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question2.

(10 Marks)

I. Let G be the grammar:

$S \rightarrow AB$

$A \rightarrow xB \mid \lambda$

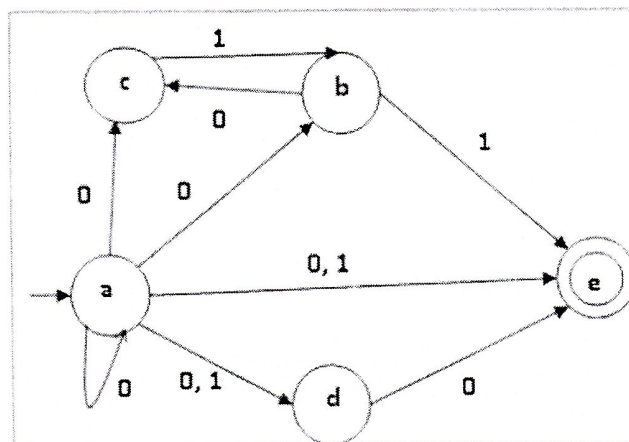
$B \rightarrow yA \mid zB$

1) What are the terminals and non-terminals of this grammar?

3) How might we generate the string $xyyA$?

4) Draw the parse tree for the previous partial derivation

II. Convert the following NFA to DFA.



Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b) grammar	c) regular expression	d) Kleene closure
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow^* \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow^* A\omega$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7 Enter
      1
     2 1 2
    3 2 1 2 3
   4 3 2 1 2 3 4
  5 4 3 2 1 2 3 4 5
 6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix. `public static boolean isMarkovMatrix(double[][] m)`
Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.
- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.
- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
- II. Java is Object-Oriented and Java is Distributed
- III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

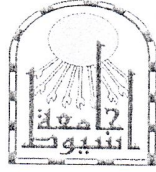
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤٤)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢٢)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤٤)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2, 4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

Question 3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question 4. Compare between the following:

(10 Marks)

1) NFA & DFA

2) Leftmost BFS/DFS

Question3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question2.

(10 Marks)

I. Let G be the grammar:

$S \rightarrow AB$

$A \rightarrow xB \mid \lambda$

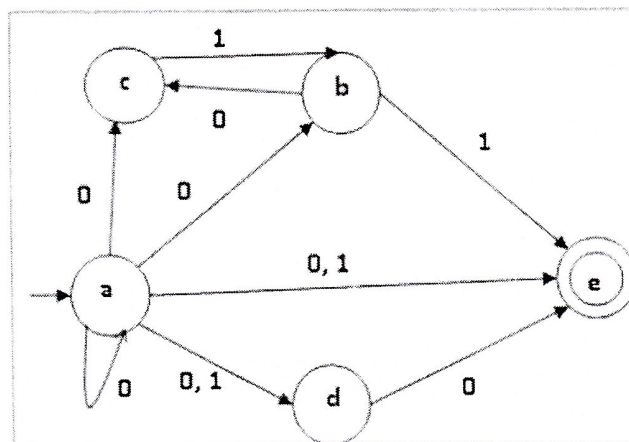
$B \rightarrow yA \mid zB$

1) What are the terminals and non-terminals of this grammar?

3) How might we generate the string $xyyA$?

4) Draw the parse tree for the previous partial derivation

II. Convert the following NFA to DFA.



Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b)	c) grammar	d) regular expression
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow * \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow * A \alpha$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix.

```
public static boolean isMarkovMatrix(double[][] m)
```

Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.

- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.

- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
II. Java is Object-Oriented and Java is Distributed
III. Increment and decrement operators.

== With My Best Wishes ==

Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$)

(6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

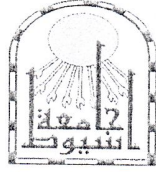
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2,4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله

3) LR(1) & SLR(1) & LALR(1)

== With My Best Wishes ==
Dr. Dalia Nashat

Question 3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question 4. Compare between the following:

(10 Marks)

1) NFA & DFA

2) Leftmost BFS/DFS

Question3. Given the following grammars:

(10 Marks)

- 1) $E \rightarrow T$
 $E \rightarrow T + E$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the Leftmost DFS Parsing algorithm to derive **int+(int + int)**

- 2) $E \rightarrow F$
 $E \rightarrow E + F$
 $F \rightarrow F * T$
 $F \rightarrow T$
 $T \rightarrow \text{int}$
 $T \rightarrow (E)$

Use the shift/reduce algorithm to derive **int + int*int + int**

Question2.

(10 Marks)

I. Let G be the grammar:

$S \rightarrow AB$

$A \rightarrow xB \mid \lambda$

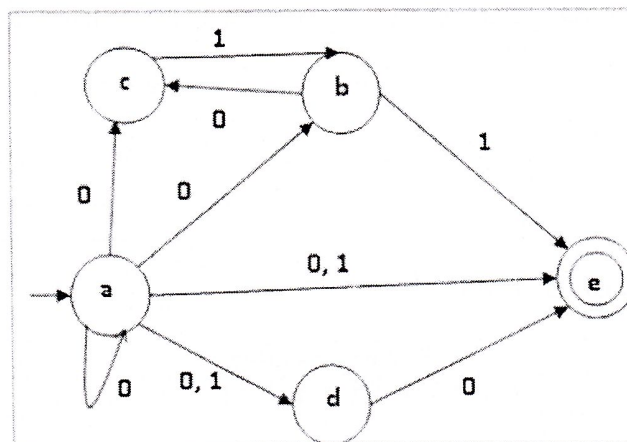
$B \rightarrow yA \mid zB$

1) What are the terminals and non-terminals of this grammar?

3) How might we generate the string $xyyA$?

4) Draw the parse tree for the previous partial derivation

II. Convert the following NFA to DFA.



Answer the following questions:

(50 Marks)

Question 1: Choose the correct answer and write it in the answer table:

(20 Marks)

1. ----- is an enumerated type representing what logical entity we read out of the source code.	a) lexeme	b) token	c) attributes	d) concatenation
2. language can be defined by using all the following except -----	a) automaton	b)	c) grammar	d) regular expression
3. $R1 \mid R2$ is a regular expression representing the ----- of $R1$ and $R2$.	a) concatenation	b) union	c) Kleene closure	d) empty set
4. The ----- transitions are followed automatically and without consuming any input.	a) 0	b) 1	c) ϵ	d) a
5. In DFA, every state must have exactly ----- transition defined for every letter.	a) one	b) two	c) three	d) four
6. ----- can be in many states at once.	a) DFA	b) NFA	c) NFA & DFA	d) NFA or DFA
7. ----- indicates a decrease in indentation.	a) NEWLINE	b) INDENT	c) DEDENT	d) SPACE
8. When parsing, our alphabet is the set of -----	a) ASCII	b) Unicode	c) Alphabets	d) Tokens
9. In CFG, capital letters at the beginning of the alphabet will represent -----	a) production rules	b) terminal	c) nonterminals	d) b and c
10. ----- analysis recover the structure described by a series of tokens.	a) Lexical	b) Syntax	c) Semantic	d) All the previous.
11. Ambiguity is a property of -----	a) language	b) parse tree	c) tokens	d) grammars.
12. A nonterminal A is said to be left recursive iff -----	a) $A \rightarrow \gamma$	b) $\alpha \Rightarrow * \beta$	c) $A \Rightarrow \alpha$	d) $A \Rightarrow * A \omega$
13. A ----- tree is a tree encoding the steps in a derivation.	a) binary	b) Read Black	c) parse	d) heap
14. ----- techniques scan the input from left-to-right.	a) Directional	b) Predictive	c) Reverse	d) CFG
15. ----- means move a terminal across the split.	a) Reduce	b) Shift	c) Predict	d) Match
16. ----- pop some number of symbols from the stack, and then push the appropriate nonterminal.	a) Reduce	b) Shift	c) Predict	d) Match
17. A ----- conflict is a state where the handle might occur but we might actually need to keep searching.	a) reduce/reduce	b) shift/shift	c) shift/reduce	d) ambiguity
18. ----- only accepts languages where the handle can be found with no right context.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
19. Any LL(1) grammar is -----.	a) LR(1)	b) SLR(1)	c) LR(2)	d) LR(0)
20. ----- LR(1) grammars are LALR(1).	a) All	b) No	c) Most	d) Few

Answers table

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer																				

18. Java was developed by -----

- a) Microsoft b) Sun Microsystems c) Oracle d) IBM

19. What will be displayed when the following code is executed?

```
int number = 6;
while (number > 0) {
    number -= 3;
    System.out.print(number + " ");
}
```

- a) 6 3 0 b) 6 3 c) 3 0 d) 3 0 -3

20. Given the following method

```
static void nPrint(String message, int n) {
    while (n > 0) {
        System.out.print(message);
        n--;
    }
}
```

What is k after invoking nPrint("A message", k)?

int k = 2;

nPrint("A message", k);

- a) 0 b) 1 c) 2 d) 3

2. Write Complete Program for the following:

(20 Marks)

- I. A program that prompts the user to enter an integer from 1 to 15 and displays a pyramid, as shown in the following sample run:

```
Enter the number of lines: 7
1
2 1 2
3 2 1 2 3
4 3 2 1 2 3 4
5 4 3 2 1 2 3 4 5
6 5 4 3 2 1 2 3 4 5 6
7 6 5 4 3 2 1 2 3 4 5 6 7
```

- II. An $n \times n$ matrix is called a positive Markov matrix if each element is positive and the sum of the elements in each column is 1. Write the following method to check whether a matrix is a Markov matrix. `public static boolean isMarkovMatrix(double[][] m)`
Write a test program that prompts the user to enter a matrix of double values and tests whether it is a Markov matrix.
- III. Twin primes are a pair of prime numbers that differ by 2. For example, 3 and 5 are twin primes, 5 and 7 are twin primes, and 11 and 13 are twin primes. Write a program to find all twin primes less than 1,000.
- IV. Use the Random class to write a program that creates a Random object with seed 1000 and displays the first 50 random integers between 0 and 100.

3. Write on the following:

(10 Marks)

- I. Machine Language, Assembly Language and High-Level Language.
- II. Java is Object-Oriented and Java is Distributed
- III. Increment and decrement operators.

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Dr. Dalila Nashat



Answer the following questions:

(50 Marks)

1. Choose the correct answer:

(20Marks)

- The extension name of a Java source code file is -----.
a) .java b) .obj c) .class d) .exe
- Which of the following lines is not a Java comment?
a) `** comments **` b) `/* comments */` c) `/* COMMENTS */` d) `/** comments */`
- Which of the following is not a reserved word?
a) classes b) void c) static d) public
- If you forget to put a closing quotation mark on a string, what kind of error will be raised?
a) logic b) runtime c) compilation d) none
- Which of the following is a valid identifier?
a) 8+9 b) 9X c) class d) \$343
- is the Java assignment operator.
a) == b) := c) = d) =:
- `Math.pow(4, 1.0 / 2)` returns -----
a) 2 b) 2.0 c) 1.0 d) 1
- Suppose `x=10` and `y=10` what is `x` after evaluating the expression `(y >= 10) || (x-- > 10)`.
a) 9 b) 10 c) 11 d) 12
- What is the number of iterations in the following loop:

```
for (inti = 1; i < n; i++) {  
    // iteration  
}
```


a) `2*n` b) `n` c) `n - 1` d) `n + 1`
- Suppose your method does not return any value, which of the following can be used as a return type?
a) void b) int c) double d) public
- Assume `double[][] x = new double[4][5]`, what are `x.length` and `x[2].length`?
a) 4 and 4 b) 4 and 5 c) 5 and 4 d) 5 and 5
- When you create an array as follow, the element values are automatically initialized to -----
`int[][] matrix = new int[5][5];`
a) 0 b) 5 c) empty d) none
- Suppose `inti= 5`, which of the following can't be used as an index for array `double[] t = new double[100]`?
a) `i` b) `(int)(Math.random() * 100)` c) `i + 10` d) `i + 6.5`
- What is the output of the following code?

```
char ch = 'F';  
if (ch >= 'A' && ch <= 'Z')  
    System.out.println(ch);
```


a) F b) f c) nothing d) F f
- The statement `System.out.printf("%5d", 123456)` outputs-----
a) 12345 b) 23456 c) 123456 d) 12345.6
- Variables that are shared by every instances of a class are----- variables.
a) public b) private c) class d) instance
- The default value for data field of a boolean type, numeric type, object type is -----, respectively.
a) true, 1, Null b) false, 0, null c) true, 0, null d) true, 1, null

Part 1 Answer ONLY one of the following two questions (15 marks):

Question 1:

1. What is Data Mining?
2. What are the KDD process?
3. What are the Data Mining Tasks?
4. What are the major tasks in data preprocessing?
5. How to Handle Noisy Data?

Question 2:

1. Classification is a two-step process. Explain this statement in details.
2. Write the basic concepts of Support Vector Machines as a linear classifier.

Part 2 Answer the following questions:

Question 3 (10 marks):

Suppose that the data for analysis includes the attribute age. The age values for the data tuples are (in increasing order) 13, 15, 16, 16, 19, 20, 20, 21, 22, 22, 25, 25, 25, 25, 30, 33, 33, 35, 35, 35, 35, 36, 40, 45, 46, 52, 70

Use smoothing by bin means to smooth the above data, using a bin depth of 3.

Question 4 (10 marks):

Suppose min. support count required is 3 and minimum confidence required is 50%. Find out the frequent itemset using Apriori algorithm and Association rules using min. support & min. confidence

TID	List of Items
T100	I1, I2, I5
T100	I2, I4
T100	I2, I3
T100	I1, I2, I4
T100	I1, I3
T100	I2, I3
T100	I1, I3
T100	I1, I2, I3, I5
T100	I1, I2, I3

Question 5 (15 marks):

Cluster the following eight points (with (x, y) representing locations) into two clusters A1(2, 10) A2(2, 5) A3(8, 4) A4(5, 8) A5(7, 5) A6(6, 4) A7(1, 2) A8(4, 9). Initial cluster centers are: A1(2, 10) and A4(5, 8). The distance function between two points $a=(x_1, y_1)$ and $b=(x_2, y_2)$ is defined as: $\rho(a, b) = |x_2 - x_1| + |y_2 - y_1|$.

Use k-means algorithm to find the three cluster centers after the second iteration.

Best Wishes
Dr Rasha Mahmoud

Answer Five questions only: (10 marks for any question)

1-a) For the joint probability density function

$$f(x, y) = x + y, 0 < x < 1, 0 < y < 1,$$

compute $f_X(x)$, $f_Y(y)$, $\rho(x, y)$, are X and Y dependent or independent.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the Gamma distribution with parameters α and β , compute the moments estimates of both α and β .

2-a) Let $X \sim N(\mu, \sigma^2)$, compute the PDF of $W = e^X$.

b) If U and V are independent chi square random variables with γ_1 and γ_2 degrees of freedom, find the PDF of $X = \frac{U/\gamma_1}{V/\gamma_2}$.

3-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the distribution with PDF

$$f(x) = e^{-(x-\theta)}, x > \theta,$$

(i) compute the maximum likelihood estimate of θ and find an unbiased estimator for θ .

(ii) construct a $100(1-\alpha)\%$ CI for the parameter θ .

b) Compute the Fisher information matrix based on a single observation from the Poisson distribution with parameter λ

4-a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the exponential distribution with parameter θ , compute the Bayes estimate of θ by assuming a gamma conjugate prior for θ based on squared error loss function.

b) Use Stirling's formula to prove that the Student T distribution with v degrees of freedom tends to the standard normal distribution $N(0,1)$ as $v \rightarrow \infty$.

5) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$:

(i) show that $T = \frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t(n-1)$, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

(ii) show that s^2 is consistent estimator for σ^2

(iii) for a random sample of size 16 with $\bar{X} = 70$ from the $N(\mu, 9)$, test the null hypothesis $H_0: \mu = 68$ against the alternative $H_1: \mu \neq 68$ at a significant level $\alpha = 0.05$.

6-a) State and prove the central limit theorem.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution $N(\theta, \sigma^2)$, compute the Bayes estimator of the parameter θ by assuming $\theta \sim N(a, b^2)$ is a conjugate prior distribution for θ when σ^2 is known under the absolute value error loss function.

Q. 3. a) Find the errors in the following code and correct them if found (6 marks)		(12 marks)
i) <code>A = zeros(4, 10); parfor i = 1:4 for j = 1:10 A(i, j) = i + j; end disp(A(i, 1)) end</code>	ii) <code>parfor i = 1:4 outputData.outArray1(i) = 1/i; outputData.outArray2(i) = i^2; end</code>	
b) Use the fact that $\pi = \int_0^1 \frac{4}{1+x^2} dx$ to approximate pi in pmode. (6 marks)		(15 marks)
Q. 4. a) If we have 1024 processors, each adds a pair of integers in 1 μ sec, What is the performance when adding two 5000-element vectors (one per processor)?(4 marks) b) What are the differences between (8 marks) I. Shared and Switched Media Interconnection Networks II. Domain decomposition and Functional decomposition III. Binary tree network and hypercube interconnection networks c) Which code executed faster and why?(3 marks)		
i) <code>x = 1:10000; xsums = cumsum(x); y = xsums(5:5:length(x));</code>	ii) <code>x = 1:10000; ylength = (length(x) - mod(length(x),5))/5; y(1:ylength) = 0; for n= 5:5:length(x) y(n/5) = sum(x(1:n)); end</code>	

Best Wishes, Dr. Hanaa A. Sayed

Answer the following questions (50 marks)

Q. 1. Complete the following sentences: •(a)..... Events or processes which occur or progress at the same time • SISD concurrent processing allowed.....(b)..... ,(c)..... •(d).....all active processor executes the same instruction synchronously, but on different data •(e)..... Largest distance between two switch nodes. • The Extend Compilers Advantages are(f)..... ,(g)..... •(h)..... A variable defined before the loop whose value is used inside the loop, but never assigned inside the loop •(i).....A task needs values from a small number of other tasks • The(j).....items used only by a single processor	(10 marks)		
Q. 2. a) After running this code, what are the type and the value of each variable? (7 marks) <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px; vertical-align: top;"> i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code> </td><td style="width: 50%; padding: 5px; vertical-align: top;"> ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code> </td></tr> </table> b) What is the Cache-coherence Problem and how the Directory-based Protocol solves it? (6 marks)	i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>	(13 marks)
i) <code>clear A</code> <code>d = 0; i = 0;</code> <code>parfor i = 1:4</code> <code>d = i*2;</code> <code>A(i) = d;</code> <code>end</code>	ii) <code>spmd(4)</code> <code>if labindex==1</code> <code>a=rand(2,2)</code> <code>else</code> <code>a=rand(1,1)</code> <code>end</code> <code>end</code>		

Department of Mathematics		قسم الرياضيات
Faculty of Science		كلية العلوم
امتحان نهائي الفصل الدراسي الثاني ٢٠١٧/٢٠١٨ م		
المستوى الرابع	الدرجة الكلية: ٥٠ درجة	تاريخ الامتحان ٢٠١٨/٥/٢٤
المقرر: (٤١٤ ر) معادلات تفاضلية جزئية		الزمن: ثلاث ساعات

أجب عن خمسة فقط مما يأتي: (١٠ درجات عن كل سؤال - بواقع ٥ درجات عن كل فقرة)

(علما بأن $p = z_x$, $q = z_y$, $r = z_{xx}$, $s = z_{xy}$, $t = z_{yy}$ هي الرموز الاصطلاحية)

١- (أ) بطريقة أويلر- أوجد الحل العام للمعادلة التفاضلية $x^2r + 2xys - xp = \frac{x^3}{y^2}$, $y \neq 0$

(ب) أثبت أن الحل الكامل للمعادلة التفاضلية $z = xp + yq + \sqrt{p^2 + q^2 + 1}$ يمثل مجموعة مستويات غلافها كرة مركزها نقطة الأصل ونصف قطرها الوحدة .

٢- (أ) أوجد حلاً للمعادلة التفاضلية $\frac{\partial}{\partial x}(x^2p) + \frac{1}{\sin y} \frac{\partial}{\partial y}(q \sin y) = 0$ على الصورة $z(x,y) = f(x) \cos y$

والذي يحقق الشروط $i) p \rightarrow 0$ as $x \rightarrow \infty$, $ii) p = -\cos y$, when $x = a$

(ب) بوضع $x = \ln X$, $y = \ln Y$ في المعادلة التفاضلية $x^2p^2 + y^2q^2 = z$ ، عين الحل الكامل والحل المفرد .

٣- (أ) بطريقة المميزات - أوجد الحل العام للمعادلة التفاضلية $r + xs - 6x^2t = x^{-1}p$, $x \neq 0$

(ب) بطريقة شاربت - عين الحل الكامل والحل المفرد (إن وجد) للمعادلة التفاضلية $q = xp + p^2$

٤- (أ) أوجد حل دالمبير لمسألة كوشي لوتر غير منتهى للمعادلة الموجية

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, t \geq 0, c \in \mathbb{R} - \{0\},$$

والتي تحقق الشروط $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ ، ثم أوجد حل هذه المعادلة عندما

$$f(x) = e^{-x^2}, g(x) = 0, c = 1$$

(ب) عين الشرط اللازم لكي يكون النظام $p_1 + p_3 = p_2 + 1$, $p_1x_1 + p_2x_2 = p_3^2$ متوافق ، ثم أوجد الحل الكامل .

٥- (أ) أوجد الحل العام للمعادلة التفاضلية $r - 2yp + y^2z = (y - 2)e^{2x+3y}$

(ب) بطريقة مونج - عين الحل الكامل للمعادلة التفاضلية $qs - pt = q^3$

٦- (أ) أوجد الحل العام للمعادلة التفاضلية $t - xq = e^{xy}$

(ب) بطريقة جاكوبي - عين الحل الكامل للمعادلة التفاضلية $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$

4-a) Derive the necessary condition for the problem

$$\text{Min } f(\underline{x}) \quad , \underline{x} \in R^n$$

$$\text{S.t } g(\underline{x}) = 0 \quad , i=1,2,\dots,m$$

Using Lagrange multiplier method

(6.5 points)

b) Use Lagrange multiplier method, to solve

$$\text{Min } f(\underline{x}) = 2x_1^2 x_2$$

$$\text{S.t } x_1^2 + 2x_1x_2 = 24$$

(6 points)

5-a) Prove that the sufficient condition for point $\underline{x}^*$, to be a minimum point for the continuous function $f(\underline{x})$, is that the Hessian matrix H evaluated at $\underline{x}^*$ is a positive definite matrix ($\underline{x}, \underline{x}^* \in R^n$) (6.5 points)

b) Find the extreme points of the Function

$$f(\underline{x}) = x_1^3 + 2x_2^3 + 3x_1^2 + 4x_2^2 + 6$$

(6 points)

With Our Best Wishes

Por. Dr. Taha Elginly

Dr. Alaa Faheem

Answer 4 Question Only From The Following:

1-a) Derive the necessary and sufficient conditions for the point x^* to be a minimum point of the function $f(\underline{x})$, $\underline{x} \in R^n$. (6.5 points)

b) Use the derived necessary and sufficient conditions to find the extreme points of the function

$$f(\underline{x}) = x_1^3 + x_2^3 + 2x_1^2 + 3x_2^2 - x_1x_2 + 2x_1 + 4x_2 \quad (6 \text{ points})$$

2-a) if the descent direction of the function

$f(\underline{x}) = 3x_1^2 + 2x_2^2 + 2x_1x_2 + 7$ at the point (1,2) is given by (-1,-1) Compute analytically the step size a to minimize this function in the given direction, and then calculate the next point. (6 points)

b) prove that the gradient vector g of the function $f(\underline{x})$ at $\underline{x}^*$ is orthogonal to the tangent plane of the surface $f(\underline{x}) = \text{constant}$ (6.5 points)

3-a) Consider the problem

$$\text{Min } f(x_1, x_2)$$

$$\text{S.t } g(x_1, x_2) = 0$$

Derive the necessary condition for $f(x_1, x_2)$ to have a minimum point at (x_1^*, x_2^*) , using the constrained variation method. (6.5 points)

b) Use the previous condition to derive the minimum point of the problem

$$\text{Min } f(x_1, x_2) = 5x_1^{-1}x_2^{-2}$$

$$\text{S.t } x_1^2 + x_2^2 - 9 = 0 \quad (6.5 \text{ points})$$

Please See Next Page

5. (a) Suppose $f \in C[a, b]$ and we want to determine a least squares approximating polynomial, that is, let $P_n(x) = \sum_{k=0}^n a_k x^k$ show that to find $P_n(x)$ the $(n + 1)$ normal equations are:

$$\sum_{k=0}^n a_k \int_a^b x^{j+k} dx = \int_a^b x^j f(x) dx, \quad j = 0, 1, \dots, n.$$

- (b) Find the least squares approximating polynomial of degree two for the function $f(x) = \cos(\pi x)$ on the interval $[0, 1]$.

6. Derive the systems arising from forward difference method and Crank-Nicolson method at any point (x_i, t_j) to the heat equation

$$\frac{\partial^2 u(x, t)}{\partial x^2} = \frac{\partial u(x, t)}{\partial t}, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = \sin \pi x, \quad x_i = ih, \quad t_j = jk.$$

انتهت الاسئلة

د. محمد / احمد حسين

د. شعبان علي بكر

Assiut University	Numerical Analysis (2)	Date: 12/5/2018
Faculty of Sciences	Code: 424 M	Time: 3 hours
Mathematics Department	B. Sc. Students in Mathematics	Grade: 50 marks

Answer 5 (five) questions ONLY from the following(grades equally distributed):

1. (a) Use Euler's method to approximate the solution of the initial-value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5, h = 0.5.$$

- (b) Solve by using the finite-difference method the boundary value problem,

$$y'' + xy' + y = 2x, \quad 0 \leq x \leq 1, y(0) = 0, y(1) = 1, h = 0.25.$$

2. (a) Suppose f is continuous and satisfies a Lipschitz condition with constant L on

$D = \{(t, y) | a \leq t \leq b, -\infty \leq y \leq \infty\}$ and that a constant M exists with

$$|y''(t)| \leq M, \text{ for all } t \in [a, b],$$

where $y(t)$ denotes the unique solution to the initial-value problem

$$y' = f(t, y), \quad a \leq t \leq b, y(a) = \alpha.$$

Let $w_0, w_1, \dots, w_N$ be the approximations generated by Euler's method for some positive integer N . Then, prove that the error bound is given by

$$|y(t_i) - w_i| \leq \frac{hM}{2L} [e^{L(t_i-a)} - 1], i = 0, 1, \dots, N.$$

- (b) Give an algorithm for the Crout factorization of the tri-diagonal linear system.

3. (a) Write the Runge-Kutta method of order four to solve the m^{th} -order system of first-order initial-value problems

$$y'_j = f_j(t, y_1, y_2, \dots, y_m), \quad a \leq t \leq b, \quad y_j(a) = \alpha_j, j = 1, 2, \dots, m,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$.

- (b) Construct the operation count for solving an $n \times n$ linear system using the Crout factorization algorithm.

4. (a) Show that the Chebyshev polynomial $T_n(x)$ of degree $n \geq 1$ has n simple zeros in $[-1, 1]$ at $\bar{x}_k = \cos\left(\frac{2k-1}{2n}\pi\right)$ for each $k = 1, 2, \dots, n$, and also, show that, $T_n(x)$ assumes

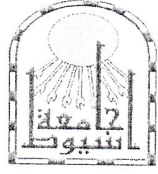
its absolute extreme for each $k = 0, 1, 2, \dots, n$ at $\bar{x}'_k = \cos\left(\frac{k\pi}{n}\right)$ with $T_n(\bar{x}'_k) = (-1)^k$.

- (b) Solve the following system

$$2x + 2y + 10z = 14, \quad 2x + 10y + z = 13, \quad 10x + y + z = 12$$

(Note: let $\mathbf{x}^{(0)} = (1.2, 0, 0)$). By using Gauss Seidel method (using three iterations only).

باقى الاسئلة فى الخلف



امتحان نهاية الفصل الدراسي الثاني ٢٠١٧-٢٠١٨

المادة: نمذجة رياضية (٤٣٤ر)

كلية العلوم - قسم الرياضيات

التاريخ: ٢٣-٥-٢٠١٨

الزمن: ٣ ساعات

(أجب عن خمسة أسئلة فقط مما يلي: (الدرجة الكلية ٥٠ درجة وكل سؤال عليه ١٠ درجات)

- ١- أذكر الخمس مراحل الأساسية للنمذجة الرياضية وناقش واحدة منها بالتفصيل. (٤د)
- ب- ناقش باختصار أنواع النماذج الرياضية وبين متى تلجأ للنموذج الفيزيائي. (٢د)
- ج- تكلم عن مصادر الخطأ في الطرق العددية وبين كيف يمكن تجنبها بقدر الامكان. (٤د)
- ٢- باستخدام طريقة المربعات الصغرى أوجد معاملات كثيرة حدود من الدرجة الأولى مرة ومن الدرجة الثانية مرة أخرى بحيث يمثلان قيم التجربة المعطاة الآتية: $(4+6=10)$ درجات

x	0.00	1.00	2.00	3.00	4.00
y	0.99	0.03	-1.02	-1.94	-3.4

- ٣- أ- عرف الشد السطحي ثم استنتج الصياغة الرياضية له ومن ثم أوجد لها لفقاعة الصابون. (٥ درجات)
- ب- أوجد النقطة على منحنى الدالة $f(x) = 4 - x^{1.5}$ والتي تجعل مربع المسافة بينها وبين النقطة $(2, 4)$ أصغر ما يمكن. (٥ درجات)
- ٤- استنتج شرط استقرار مانع ثقيل فوق مانع خفيف تحت تأثير عجلة الجاذبية الأرضية مبينا أهمية هذه المسألة من ناحية الطاقة (مسألة رايلي - تايلور للاستقرار. (١٠ درجات)
- ٥- أ- أذكر ما تعرفه عن: (الأمثلية - نمذجة الأنظمة الديناميكية - المحاكاة) مبينا مدي التقارب أو التباعد بينهم. (٤ درجات)

ب- استخدم طريقة K-B لإيجاد الحل التقريبي للنظام الفيزيائي التذبذبي $\ddot{\theta} + \omega^2 \theta = \varepsilon \dot{\theta}$

عندما يكون التردد الطبيعي للنظام ثابتا (٦ درجات)

٦- دائرة كهربية تحتوي على مكثف C وملف حثي L وقوة دافعة كهربية $E(t)$ مقدارها

$100 \sin \alpha t$ إذا كانت سعة المكثف $C=0.1$ و $L=0.1$ و $q(0)=0$ و $I(0)=0$ أوجد الشحنات على المكثف والتيار في الدائرة. (٥ درجات)

ب- من مفهوم النمذجة الرياضية استنتج الصياغة الرياضية لمعادلة لابلاس. (٥ درجات)

راجع أ. د. جمال مختار محمود

أ. د. محمود حامد عبيد الله