جامعة أسيوط	بسم الله الرحمن الرحيم	دور سبتمبر 2015 م
قسم الرياضيات		الزمن: ثلاث ساعات
المادة: معادلات تفاضلية عادية و جزئية		الفرقة: تمهيدي ماجستير (رياضيات)

أجب عن خمسة فقط من الأسئلة التالية [يخصص لتقدير السؤال 20 درجة و لجزء السؤال 10 درجات، ما عدا السؤالين الأول و الثاني فيخصص للجزء الأول 6 درجات و للجزء الثاني 14 درجة]:

1. a) If $\underline{f}(t,\underline{x}) \in C^1(B_0)$, where B_0 is any closed bounded set in $\mathbb{R}^{n+1}$, then there exists a positive constant L such that $\underline{f}$ satisfies the Lipschitz condition:

$$\|\underline{f}(t,\underline{x}) - \underline{f}(t,\underline{y})\| \le L\|\underline{x} - \underline{y}\|, \text{ in } B_0.$$

b) Assume that $\underline{f}(t,\underline{x})$ is continuous on $B_0 : t_0 \le t \le t_0 + a, \|\underline{x} - \underline{x}_0\| \le b$, where a and b are positive real numbers, and $\underline{f}$ satisfies the Lipschitz condition in B_0. Let $M = \max\limits_{(t,\underline{x})\in B_0} \|\underline{f}(t,\underline{x})\|, \alpha = \min(a, b/M)$, then the initial value problem $\underline{x}' = \underline{f}(t,\underline{x}), \underline{x}(t_0) = \underline{x}_0$ has a unique solution $\underline{x}(t)$ on $[t_0, t_0 + \alpha]$.

2. a) The solution $\underline{x}(t)$ of $\underline{x}' = A(t)\underline{x} + \underline{B}(t)$ satisfying $\underline{x}(t_0) = \underline{x}_0, t_0 \in (r_1, r_2)$ is given by:

$$\underline{x}(t) = \phi(t)\underline{x}_0 + \int_{t_0}^{t} \phi(t)\phi^{-1}(s)\underline{B}(s)ds, \; r_1 \le t \le r_2,$$

where $\phi(t)$ is a fundamental matrix of $\underline{x}' = A(t)\underline{x}$ satisfying $\phi(t_0) = I$.

b) Apply the variation of constant formula to obtain the solution of the system:

$$x_1' = 2x_1 - x_2 + 1, \; x_2' = 16x_1 - 6x_2 + t^2,$$

satisfying the initial conditions $x_1(0) = 1, x_2(0) = 0$.

3. a) Prove that the solution of the system:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} -5 & 4 & 0 \\ 0 & 0 & 3 \\ -12 & 8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

approaches zero as $t \to \infty$.

b) Let $\underline{x} \in \mathbb{R}^n$, A is an $n \times n$ constant matrix, and $B(t)$ is an $n \times n$ continuous matrix on $0 \le t < \infty$. If all the characteristic roots of A have negative real parts, then all the solutions of $\underline{x}' = (A + B(t))\underline{x}$ approach zero as $t \to \infty$, provided $\|B(t)\| \to 0$ as $t \to \infty$ holds.

P.T.O. إقلب الصفحة من فضلك

4. a) By Charpit's method find the complete integral of the nonlinear equation:

$p^2x + q^2y = z$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

b) Apply Jacobi's method to obtain the complete solution of the nonlinear equation:

$(p_1 + x_1)^2 + (p_2 + x_2)^2 + (p_3 + x_3)^2 = 3(x_1 + x_2 + x_3)$, where $p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}, p_3 = \frac{\partial z}{\partial x_3}$.

5. a) Use the method of Laplace to find the general solution of the linear equation:

$$xy\frac{\partial^2 z}{\partial x^2} + (y^2 - x^2)\frac{\partial^2 z}{\partial x\,\partial y} - xy\frac{\partial^2 z}{\partial y^2} + y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 2(x^2 - y^2).$$

b) By applying Monge's method obtain the general solution of the nonlinear equation:

$$3z_{xx} + z_{xy} + z_{yy} + (z_{xx}z_{yy} - z_{xy}^2) = -9.$$

6. a) Obtain common complete integral of the nonlinear simultaneous partial differential equations:

$$p_1x_1 + p_2x_2 - p_3^2 = 0,\ p_1 - p_2 + p_3 - 1 = 0.$$

b) Find a solution $u(x,t)$ of the equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\quad x \geq 0, t \geq 0,$$

which satisfies the conditions:

$$u(x,0) = 2\sin 3\pi x,\ u(0,t) = u(1,t) = 0.$$

مع أخلص التمنيات بالتوفيق ،، أ.د. أحمد أبو العلا ، أ.د. عبد الرحيم صادق

Dept. of Math.	M.Sc. EXAM. Biostatistics (643R)	Sept., 14, 2015 Time: 3 H

ALL QUESTIONS ARE TO BE ATTEMPTED(20 points for every question)

Q (1): The following are the activity values (micro moles per minute per gram of tissue) of a certain enzyme measured in the normal gastric tissue of 15 patients with gastric carcinoma: 0.36, 1.19, 0.61, 0.798, 0.27, 2.46, 0.57, 1.83, 0.54, 0.37, 0.45, 0.26, 0.45, 0.97, 0.37,

(i) Calculate the mean $\overline{X}$, variance S^2 and CV.

(ii) Construct 90%, 95% and 99% confidence intervals for the population mean μ (normal population)

(iii) Explain why the three intervals that you construct are not of equal width.

(iv) Indicate which of the previous intervals prefer to use as an estimate of the population mean, and state the reason for your choice. ($t_{14,0.05} = 1.76, t_{14,0.025} = 2.15, t_{14,0.005} = 2.98$)

Q (2): (a) (i) Define: Estimation of a parameter- Standard deviation and standard error- Kendall's rank correlation

(ii) The following table gives the values for the birth weight (x) and the increase in weight between days 70 and 100 of life, expressed as a percentage of the birth weight (y) for 12 infants.

X	112	111	107	119	92	80	81	84	118	106	103	94
Y	63	66	72	52	75	118	120	114	42	72	90	91

Calculate Kendall's rank correlation.

(b) (i) Write the 95% confidence intervals for the population proportion π in terms of sample proportion p considering large sample size, and for the difference for proportions $\pi_1 - \pi_2$ in terms of $p_1 - p_2$, considering large sample sizes.

(ii) A study was conducted to look at the effects of oral contraceptives (OC) on heart disease in women 40–44 years of age. It is found that among $n_1 = 50$ current OC users, 13 develop a myocardial infarction (MI) over a three-year period, while among $n_2 = 100$ non-OC users, seven develop an MI over a three-year period. Construct the 95% confidence intervals for the MI rates π_1 and $\pi_1 - \pi_2$, use $Z_{0.025} = 1.96$.

Abd El-Baset, A. Ahmed

Q (3): (a) Suppose that the regression line of y on x is $y = \beta_0 + \beta_1 x$ and the regression line of x on y is $x = \delta_0 + \delta_1 y$, derive the estimates $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\delta}_0, \hat{\delta}_1$, then show that $r^2 = \hat{\beta}_1 \hat{\delta}_1$.

(b) Suppose that we choose a sample of size n from a normal population with mean μ. Let $S^2 = 16, \sum_{i=1}^{n} x_i^2 = 84$ and $\sum_{i=1}^{n} (x_i - \bar{x})^2 = 64$. Show that $n = 5$ and $\bar{x} = 2$, then find the 95% confidence interval for the population mean, use $t_{4,0.025} = 2.776$.

Q (4): (a) The following are the heights and weights of 10 women as in the following table

Height	152	156	158	160	162	162	164	164	166	166
Weight	52	50	47	48	52	55	55	56	60	60

Calculate Spearman's rank correlation coefficient of height and weight.

(b) To assess whether the level of iron in the blood is the same for children with cystic fibrosis as for healthy children, a random sample is selected from each population. The $n_1 = 9$ healthy children have average serum iron level $\bar{X}_1 = 18.9$ μmol/l and standard deviation $S_1 = 5.9$ μmol/l. The $n_2 = 13$ children with cystic fibrosis have average iron level $\bar{X}_2 = 11.9$ μmol/l with sample standard deviation $S_2 = 6.3$ μmol/l. Consider normal populations and equal variances. Is there a true difference in population means ? Test at the $\alpha = 0.05$ level. (use $t_{20,0.025} = 2.086$)

Q (5): (a) (i) Define: Rejection region, Level of significance.

(ii) In the case of testing the null hypothesis $H_0 : \mu_1 = \mu_2$ write the steps that you do to make a decision regarding this hypothesis.

(b) Six healthy sheep were injected with an antibiotic. Their mean blood serum concentrations of the antibiotic 1.5 hours after injection were of interest.

i) Assuming a 99% confidence interval was [21.11, 36.22], what are the sample mean and sample standard deviation?

ii) With the sample mean and sample standard deviation above, what confidence level does the interval [23.85, 33.48] correspond to? (use $t_{5,0.005} = 4.032$, $t_{5,0.4875} = 2.57$)

Best wishes ,,, **Prof. Dr. Abd EL-Baset A. Ahmad**

Abd EL-Baset A. Ahmad

University of Assiut
Faculty of Science
Department of Mathematics

Final Exam (641): Statistics

Department of Chemistry: Master Preparatory Year 2014 – 2015	Date: September 16, 2015
Time: 3 hour	Degree: 100 marks

Answer only five questions from the following:

[20 marks is the degree of each question.]

1) What does it mean by:

p-value - Type I error - Level of significance - Power of the test.

2) From clinical trials, it is assume that 30 % of mice inoculated with a serum will not develop protection against a certain disease. If 5 mice were inoculated, find

a. The probability of at least 4 mice will contract the disease.

b. The probability that exactly 3 mice will contract the disease.

c. The mean and variance of mice will contract the disease.

3-a) We rolled a normal die 120 times, and got the following outcomes

Category	1	2	3	4	5	6
Observed	30	18	24	20	16	12

Based on these results, what would you conclude about the claim that the dice is fair. Test at 5 % significant level.

3-b) In a certain experiment it was required to estimate the nitrogen content of the blood plasma of a certain colony of rates at their 35^{th} day of age. A sample of 9 rats was taken at random and the following data was obtained (grams per 100 c.c. of plasma): 0.98, 0.83, 0.99, 0.86, 0.90, 0.81, 0.94, 0.92, and 0.87. Find the estimates for the average nitrogen content and the variation in nitrogen content in the colony.

4-a) In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let p be the proportion of homes in this city that are heated by oil. Construct a 95 % confidence interval on the true proportion p.

4-b) Suppose that an investigator wishes to estimate the content of mercury in the water of a certain area. He collected 13 measurements at random and obtained the following data (as parts per million, ppm): 409, 403, 402, 406, 401, 403, 389, 403, 407, 402, 410, 405, and 399. Based on these data, construct a 95 % confidence interval for the content of mercury.

5 A researcher was interested in comparing the mean of two populations, A and B, based on the following two independent samples:

Observations from population A: 89, 82, 79.0, 80, 82, 81,
Observations from population B: 83, 84, 85, 80, 78, 72, 77,

Assuming non-pooled varances, construct a 95 % confidence interval for the difference between the two means, $\mu_A - \mu_B$.

(Please turn the page for the rest of the questions) %

6) The following table shows the relation between two variables X and Y.

X	3	5	4	4	2	3
Y	86	95	92	83	78	82

a. Calculate Pearson's correlation coefficient between X and Y.

b. Find the best regression line of Y on X.

Use the suitable tabulated values of the following values:
$T_{[0.975;\ 13]}=2.1604$, $T_{[0.975;\ 12]}=2.1788$, $T_{[0.975;\ 11]}=2.201$; $Z_{0.975}=1.96$, $Z_{0.995}=2.575$,
$\Pr(0<Z<2.29)=0.4890$, $\Pr(0<Z<1.14)=0.3729$, $Z\sim N(0,1)$;
$\Pr(0<X<11.071)=0.95$, $\Pr(0<X<12.83)=0.975$, $X\sim\chi^2_5$.

With the best of luck,

Department of Mathematics

	FACULTY OF SCIENCE ASSIUT UNIVERSITY	

Final Exam on Engineering Geophysics Course (643 G) for Graduate Students

September 2015 Time: 3 hours

1- Answer only <u>two</u> questions of the following:

a) Discuss the protocol that is commonly used for the selection of appropriate geophysical tools to study a geological or engineering problem with appropriate example.

b) According to your readings, write on some concluding remarks and recommendations for a good practice of engineering geophysics

c) Write in details how to design successful ground penetrating radar (GPR) survey.

2- Write in brief about <u>four</u> of the following:

a) The geological applications of engineering geophysics

b) Geotechnical properties derived from geophysical properties

c) The different resistivity field techniques

d) Different sources of Self-Potential method

e) Most important corrections applied to gravity data

f) Problems associated with magnetic data interpretation

g) Different sources of GPR attenuation

End of questions Good luck

Associate Professor: Gamal Zidan AbdelAal

Assiut University	**Examination For Post Graduate**	**5^{th} September. 2015**
Mathematics Department	**Students (MSc) :General Relativity (622 MATH)**	**Time allowed:3 hours**

Full mark (100) is given for the answer of FIVE questions : (20 for each)

1.a) Define the tangent space $T_P(M)$ and its dual $T^*_P(M)$ of an n-dimensional differentiable manifold M at P.

1.b) Define tensor field $(T^r{}_s)_P(M)$ and its dual $(T^{*}{}^r{}_s)_P(M)$.

2.a) Discuss parallelism of vectors on a manifold . Define the covariant and absolute derivative of a vector field λ^a.

2.b) Prove that the covariant derivative of the fundamental tensor of $g_{ab}=0$.

3.a) Define geodesic curves in an n-dimensional differentiable manifold and show that are the solutions of the differential equations :

$$\frac{d^2u^i}{dt^2}+\Gamma^i_{jk}\frac{du^j}{dt}\frac{du^k}{dt}=0 \ ,$$

where t is an affine parameter along the geodesic curves $u^i(t)$.

3.b) Show that, for a given Lagrangian : $L(\dot{x}^c, x^c)=\frac{1}{2}g_{ab}(x^c)\dot{x}^a\dot{x}^b$, the Eular-Lagrangian equations : $\frac{d}{du}\left(\frac{\partial L}{\partial \dot{x}^c}\right)-\frac{\partial L}{\partial x^c}=0$ reduce to of the geodesic equations .

4.a) Let $T^{ab}=(\rho+p)u^a u^b-p\eta^{ab}$ be the energy Momentum tensor for a perfect fluid has pressure p and density ρ , using $T^{ab}{}_{,b}=0$ to prove that :

$u^a{}_{,b}u_a=0$ which implies that $u^a{}_{,b}u_a=0$

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4.b) Define the Riemann curvature tensor $R^{a}{}_{bcd}$ and R_{abcd} and state its orperties.

Prove that the Einstein tensor : $G_{ab} = R_{ab} - ½\, \mathrm{R}\, g_{ab}$ is trace free , i.e. $G^{ab}{}_{;b} = 0$.

5.a) Using the Einstein equations : $R^{ab} - ½\, R\, g^{ab} = \kappa T^{ab}$ to show that : $R = -\kappa T$,

where $T = T^{a}{}_{a}$.

5.b) Comparing Einstein equations with Poisson equations to calculate the coupling constant κ .

6.a) Discuss the properties of the Schwarzschild solution :

$$d\tau^2 = (1 - 2mG/r)dt^2 - (1 - 2mG/r)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2).$$

6.b) Discuss length and time in the Schwarzschild geometry.

Prof.M.Abdel-Megied

Assiut University
2014/2015

Final Exam Graduate Students (Pre-Mater Degree)
Mathematics and Computer Science
Artificial Intelligence M656
3 Hours – 100 Points

Faculty of Science
Mathematics Dept.

Answer the Following Questions:

Notes: **GAs**: Genetic Algorithms, **GP**: Genetic Programming, **SA**: Simulated Annealing, **EAs**: Evolutionary Algorithms, **NN**: Neural Network.

Question One: (20 Points)

A. Complete the following statements: (**2 Point each**)

1. In a tree generated by the GP algorithm, internal nodes are called ___.
2. In GAs, the ___ consists of a number of genes based on the problem at hand.
3. The main difference between GAs and GP lies is ___.
4. Values of a good probability function in SA must ____when temperature decreases.
5. In the ___ selection strategy, an individual are selected according to its performance compared with all individuals in the population.

B. Choose an appropriate answer for each of the following items: (**1 Point each**)

1. At the maximum temperature in the SA algorithm, worse moves ___. (almost accepted, maybe accepted, almost rejected)
2. ___ describes the genetic composition of an individual in EAs as inherited from its parents. (Genotypes, Phenotypes, Prototypes)
3. Using crossover operator for parent xxxxxxx and parent yyyyyyy, we cannot get ___. (xxyyyxx, xxxxxxy, xxxxyyyy)
4. ___ used in SA to balance between intensification and diversification searches. (Maximum temperature, Minimum temperature, Cooling schedule)
5. When neurons in a single field connect back onto themselves the resulting network is called a/an ___ NN.

C. Put ***True*** or ***False*** for each of the following items, and correct the false ones: (**1 Point each**)

1. GAs maintains several possible solutions, whereas SA works with one solution.
2. GP evolves individuals represented as trees of fixed length.
3. In the Recurrent NN, the network has no loops.
4. Classical optimization uses derivatives information; however, EAs uses no derivative information.
5. The main concept of evolutionary computing is "survival of the week: the fittest must die".

Question Two: (20 Points)

A. Compare between each pairs of the following items: (**8 Points**)

1. ***Linear Strategy / Geometric Strategy*** as the temperature decrement in SA.
2. ***Intersection/ Union*** of two fuzzy sets.
3. ***Supervised/ Unsupervised*** Learning.
4. ***Elman / Jordan*** Simple Recurrent NN.

B. Explain, in details, the following terms in EAs: (**12 Points**)

Initial population – Fitness function – One-point crossover – Random mutation.

Please flip the paper

بسم الله الرحمن الرحيم

كلية العلوم - قسم الرياضيات

امتحان تمهيدي ماجستير رياضيات

اسم المقرر ورقمه : 608 ر جبر خطي

الزمن : ثلاث ساعات

تاريخ الامتحان: الاثنين 7 / 9 / 2015 م

أجب عن **خمسة فقط** من الأسئلة التالية :- (**عشرون درجة لكل سؤال**)

1- a) Let $\oplus$ and $\odot$ be vector sum and scalar multiplication defined on $\mathbb{R}^2$ as follows: $(x_1,x_2)\oplus(y_1,y_2)=(x_1+2y_1,3x_2-y_2)$,
$c\odot(x_1,x_2)=(cx_1,cx_2)$. Decide if $\mathbb{R}^2$ with these two operations is a vector space. If this is not the case, state which of the axioms fail to hold. **(8 Marks)**

b) Let $V=\mathbb{R}^3$ and $W=\{(a_1,a_2,a_3):(a_1+a_2+a_3)^2=0\}$. Determine if W is a subspace of V. **(4 Marks)**

c) Let V be a vector space and S be any nonempty subset of V. Prove that $\operatorname{Span}(S)$ is a subspace of V. **(8 Marks)**

2- a) Let $W_1=\operatorname{Span}(S_1)$ and $W_2=\operatorname{Span}(S_2)$ be subspaces of a vector space V, prove that $W_1+W_2=\operatorname{Span}(S_1\cup S_2)$. **(8 Marks)**

b) Let $v,w\in V$. Show that $\{v,w\}$ is linearly independent if and only if $\{v+w,v-w\}$ is linearly independent. **(4 Marks)**

c) Let V be a vector space and S be a nonempty subset of V. Prove that S is a basis of V if and only if every vector $x\in V$ may be written uniquely as a linear combination of the vectors in S. **(8 Marks)**

3- a) Prove that a function $T:V\to W$ is a linear transformation if and only if for all $a,b\in\mathbb{R}$ and all $u,v\in V$, $T(au+bv)=aT(u)+bT(v)$. **(6 Marks)**

b) Let $T:V\to W$ be a linear transformation and V be a finite dimensional vector space. Prove that T is uniquely determined by its value on the members of a basis of V. **(7 Marks)**

c) Determine whether the function $T:\mathbb{R}^2\to\mathbb{R}^4$ defined by $T((a_1,a_2))=(a_2,a_1,a_1,a_2)$ is a linear transformation. **(7 Marks)**

4- a) Let $T:V\to W$ be a linear transformation where $\dim(V)=m$ and $\dim(W)=n$. Define the matrix of T with respect to a basis of V and a basis of W. **(6 Marks)**

أنظر بقية الأسئلة في الخلف

b) Let $T: V \to W$ be a linear transformation between a vector space V of dimension k and a vector space W of dimension l. Let $\alpha = \{v_1, v_2, \ldots, v_k\}$ be a basis for V and $\beta = \{w_1, w_2, \ldots, w_l\}$ be a basis for W. Prove that for each $v \in V$, $[T(v)]_\beta = [T]_\alpha^\beta [v]_\alpha$.

(6 Marks)

c) Let $T: V \to W$ be a linear transformation. Define the kernel of T, $Ker(T)$, and prove that $Ker(T)$ is a subspace of V. **(6 Marks)**

d) Find the kernel of $T: \mathbb{R}^3 \to \mathbb{R}$ defined by
$T((x_1, x_2, x_3)) = 2x_1 - x_2 + 3x_3$. **(2 Marks)**

5- a) Define the image, $\mathrm{Im}(T)$, of a linear transformation $T: V \to W$ and prove that $\mathrm{Im}(T)$ is a subspace of W. **(7 Marks)**

b) Let V be a finite dimensional space and $T: V \to W$ be a linear transformation. Prove that $\dim(Ker(T)) + \dim(\mathrm{Im}(T)) = \dim(V)$.

(6 Marks)

c) Prove that a linear transformation $T: V \to W$ is injective if and only if $\dim(Ker(T)) = 0$. **(7 Marks)**

6- a) Let $T: V \to W$ be a linear transformation. Prove that a vector x is an eigenvector of T with eigenvalue λ if and only if $x \neq 0$ and $x \in Ker(T - \lambda I)$. **(7 Marks)**

b) Prove that similar matrices have equal characteristic polynomials.

(7 Marks)

c) Show that if A is diagonalizable, then A^k is diagonalizable for all $k \geq 2$. **(6 Marks)**

مع أطيب تمنياتنا لكم بالتوفيق و النجاح

أ.د/ فتحي هشام خضر

انتهت الأسئلة

Dept. of Mathematics
Faculty of Science
Assiut University
Sep. 7, 2015

Course: Functional Analysis
Code: 610 M
M.Sc. Exam
Time: 3 Hour

Answer five questions only from the following questions:

First Question (20 Degree)

(a) Define the Hilbert space and prove that the space of all continuous functions C[-1,1] with respect the inner product : $< f, g >= \int_{-1}^{1} f(t)\,\overline{g(t)}\,dt$, $f, g \in C[-1,1]$

is not a Hilbert space.

(b) State and prove Hahn Banach theorem for normed space.

(c) Give an example to show that every self adjoint operator is normal but the converse is not true.

Second Question (20 Degree)

(a) State and prove spectral mapping theorem for polynomial.

(b) Define strong convergence and weak convergence and show that weak convergence does not imply strong convergence.

(c) If a normed space X is reflexive . Show that X' is a reflexive.

Third Question (20 Degree)

(a) Let (X,d) be a compact metric space and $T: X \to X$ be a contractive mapping .Show that T has a unique fixed point.

(b) State and prove the Banach contraction mapping theorem in a metric space.

(c) Use the Banach contraction mapping theorem to show that the differential equation: $\frac{df}{dx} = (f(x) + x)$, $0 \le x \le 1$, $f(0) = 0$,

has a unique solution.

Fourth Question (20 Degree)

(a) Let $F: R^n \to R^n$ be given by $F(x_1, x_2, \ldots, x_n) = (y_1, y_2, \ldots, y_n)$ where:

$$y_i = \sum a_{ij} x_j + b_i \quad , i = 1, 2, \ldots, n.$$

Show that F is a contraction map in the sup. metric with contraction ratio λ if $\sum |a_{ij}| \le \lambda < 1$

(b) Using (a) to deduce that F(x)=x has a solution.

(c) *Let $T \in B(X, X)$, where X is a Banach space. If $\|T\| \le 1$, then $(I - T)^{-1}$ exists as a bounded* linear operator on the whole space X and $(I - T)^{-1} = \sum_{i=0}^{\infty} T^i = I + T + T^2 + \ldots\ldots$

Fifth Question (20 Degree)

(a) Prove that the resolvent set $\rho(T)$ of a bounded linear operator T on a complex Banach space X is open and the spectrum $\sigma(T)$ is closed.

(b) If z is a fixed element of an inner product space X, show that

$$f(x) = < x, z >,$$

defines a bounded linear functional f on X of norm $\|z\|$.

(c)Show that if $T: H \to H$ is a bounded self-adjoint linear operator on a Hilbert space H so is T^n, $n \in Z^+$.

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Six Question (20 Degree)

(a) State and prove the uniform boundedness principle for a family of operators from a Banach space X to a normed space Y and show that the principle is not valid if X is only a normed space.

(b) Use the uniform boundedness principle to prove that there exists real valued continuous functions whose Fourier Series diverge at a given point t_0.

(c) Let X be a Banach space ,Y a normed space and $T_n \in B(X,Y)$ such that $(T_n x)$ is Cauchy in Y for every $x \in X$. Show that $\|T_n\|$ is bounded.

Prof.R.A.Rashwan

The End

Assiut University
Faculty of Science
Mathematics Dept.

Master Level Exam
Optional Course 659M
Tensor Analysis

2014/2015
Time: 3 Hours

Answer the following questions:

Question No. 1

(a) If x^i is the coordinate of a point in n-dimensional space, and $\phi(x^1, x^2, ..., x^n)$ is a scalar function, show that dx^i , $\frac{\partial \phi}{\partial x^i}$ are tensors, $\frac{\partial \phi}{\partial x^i} dx^i$ is invariant.

(b) Show that there is no distinction between covariant and contravariant vectors when we restrict ourselves in rectangular Cartesian transformation of coordinates.

Question No. 2

(a) If T_τ is the component of a covariant vector, show that $\left(\frac{\partial T_i}{\partial x^j} - \frac{\partial T_j}{\partial x^i} \right)$ are component of a skew symmetric tensor of rank two.

(b) If the tensors a_{ij} and g_{ij} are symmetric, u^i and v^i are the components of contravariant vectors satisfying the equations.

$(a_{ij} - kg_{ij})u^i = 0$, $(a_{ij} - k'g_{ij})v^i = 0$

where $i, j = 1,2,...,n$, $k \neq k'$

Prove that $a_{ij}u^i v^i = 0$ and $g_{ij}u^i v^i = 0$

Question No. 3

(a) Prove that the metric tensor g_{ij} is a covariant symmetry tensor of rank two and $g_{ij}dx^i dx^j$ is an invariant.

(b) Find the metric and component of first and second fundamental tensors in the cylindrical coordinates.

P.T.O.

Question No. 4

(a) Define Christoffel's symbols $[ij,k]$ and $\begin{Bmatrix} i \\ jk \end{Bmatrix}$ and calculate

$[ik\,,\,j] + [jk\,,i], \begin{Bmatrix} i \\ ij \end{Bmatrix}, \begin{Bmatrix} i \\ ii \end{Bmatrix}$.

(b) For the metric $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \; d\phi^2$, find the values of

$[2\;2{,}1], [1\;3{,}3], \begin{Bmatrix} 1 \\ 22 \end{Bmatrix}$, and $\begin{Bmatrix} 3 \\ 13 \end{Bmatrix}$.

Question No. 5

(a) Find Gradient, Divergence, and Curl, $\nabla^2\phi$ in tensor forms where ϕ is a scalar function.

(b) For a V_2 referred to an orthogonal system of parametric curves, show that the component of a Ricci tensor satisfy

$R_{12} = 0, R_{11}g_{22} = R_{22}g_{11} = R_{1221}, Rg_{11}g_{22} = 2R_{1221}, 2R_{ij} = Rg_{ij}$.

Question No. 6

(a) Prove that the differential equations of a geodesic curve in V_n are

$$\frac{d^2x^m}{ds^2} + \begin{Bmatrix} m \\ jk \end{Bmatrix} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0.$$

(b) Prove that the necessary and sufficient condition that the hypersurface $\phi =$ const. form a system of parallel is that $(\nabla\phi)^2 = 1$.

Good Luck

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Prof. Mohamed Abd El-latif Soliman

Assiut University Examination For Post Graduate 12^{th} September. 2015

Mathematics Students (MSc) :Special course

(Geometry of Lorentz-Minkowski space)

Department (622 MATH) Time allowed:3 hours

Full mark (100) is given for the answer of FOUR questions : (25 for each)

1.a) Define the Lorentz-Minkowski space E_1^3 . Prove the following:

(i) Let $v \in E_1^3$, then v is time like vector if and only if $<v>^{\perp}$ is a space like and so $E_1^3 = <v> \oplus <v>^{\perp}$. For space like vector v we have v is space like if and only if $<v>^{\perp}$ is time like .

(ii) $U \subset V$ be a subspace of E_1^3 , then U is a space like and $U^{\perp}$ is time like.

(iii) If U is a subspace , then U is light like if and only if $U^{\perp}$ is light like .

1.b) Let u and v be two light like vectors , prove that they are dependent linearly if and only if $<u,v> = 0$. Let $U \subset E_1^3$ be a 2-dimensional subspace , prove the following statement are equivalent : (i) U is a time like subspace . (ii) U contains two independent linearly light like vectors . (iii) U contains a time like vector.

2.a) Define the time like cone $\mathcal{T}$ at a point $u \in E_1^3$. Prove that for any two time like vectors u and v lie in the same time like cone if and only if : (i) $<u,v> < 0$. (ii) $u \in \mathcal{T}(v)$ if and only if $\mathcal{T}(u) = \mathcal{T}(v)$. (iii) the time like cones are convex.

2.b) Let u and v two time like vectors , prove that $| <u,v> | \geq \sqrt{-<u,u>} \sqrt{-<v,v>} |$ and define the angle between u and v .

3.a) Define the Frenet frame for a curve α parametrised by the pseudo-lengh-arc in the ceses:

(i) α be a space like , (ii) α be time like.

P . T . O . >>>>>>>>>>>

3.b) Let $\kappa : \mathcal{I} \dashrightarrow \mathbb{R}$ be a smooth function and let $\mathcal{P}$ be a time like plane , then there exist unique space like curve in $\mathcal{P}$ with the curvature κ . The same result holds to assign the existence of a time like curve in $\mathcal{P}$ with κ as curvature function.

4.a) Define the space like and time like surfaces in E_1^3 . Give examples for these two types of surfaces. Write the second fundamental form for a time like surface $M \in E_1^3$ to get a formula for the Gaussian curvature $\mathcal{K}$ and the mean curvature $\mathcal{H}$.

4.b) Define the Weingarten endomorphism $A := A_N$ of the immersion $x : M \to E_1^3$. where N is the Gauss map , Give a formula for $\mathcal{K}$ and $\mathcal{H}$.

5.a) Discuss the ruled surface as a time like surface whose Weingarten endomorphism is not diagonalizable .

5.b) Show that the hyperbolic plane $\mathbb{H}(r, p_0) = \{p \in E_1^3 : (p - p_0),(p - p_0) = -r^2, z > 0\}$ is totally umbilic . If α (s) = (cosh s) p + (sinh s) v , then prove that α (s) $\in \mathbb{H}$ is a geodesic.

Prof.M.Abdel-Megied

Department of Mathematics,
Faculty of Science,
University of Assiut .
Year: 2014-2015

اسم المقرر: تحليل عددي | الدرجة الكلية: ١٠٠ درجة | الزمن: ثلاث ساعات
رقم المقرر: ٦٠٧ر | ٢٠ درجات لكل سؤال | تاريخ الإمتحان: ٢٠١٥/٩/٥م

أجـــب عن خمســـة من الأسئلة التالية:

1- Assume $g \in C^1[a,b],\ a \le g(x) \le b\ \forall x \in [a,b]$ **and** $max_{[a,b]} |g'(x)| < 1$.

Prove:

i) The convergence of the iteration $x_{n+1} = g(x_n), n \ge 0$ **for any** $x_0 \in [a,b]$ **to the fixed point** α **of** $g(x)$. **(7 marks)**

ii) $\lim_{n\to\infty} \frac{\alpha - x_{n+1}}{\alpha - x_n} = g'(\alpha)$. **(7 marks)**

iii) Give an algorithm for Aitken's process to accelerate linear convergence. (6 marks)

2- For the solution of system of nonlinear equations. Give an algorithm for each of the following:

i) The Steepest descent method. (8 marks)

ii) Newton's method. (8 marks)

iii) Comment on the order of convergence of each method. (4 marks)

3- i) Derive Lagrange's interpolation formula. (10 marks)

ii) Given $f(x)$ **and** $f'(x)$ **at** $x = a$ **and** $x = b$ **. Construct the Cubic Hermite interpolation polynomial for** $f(x)$ **. Also find the interpolation error. (10 marks)**

4- i) Give an algorithm for a near minimax approximation for a given function. (7 marks)

ii) Give a formula for asymptotic error estimate for approximating $I(f) = \int_a^b f(x)dx$ **using Trapezoidal rule. Hence give a formula for a corrected trapezoidal rule. (7 marks)**

iii) Let T_h **and** T_{hl2} **be the trapezoidal approximations for** $I(f)$ **with** h **and** $h/2$ **respectively. Apply Richardson's extrapolation to get an improved value. (6 marks)**

5- i) Give the general iteration scheme for solving system of linear equations and discuss its convergence. (10 marks)

ii) Apply the QR factorization of an $m \times n$ **when** $m > n$ **and the columns of** A **are linearly independent, to obtain the least-squares solution of the over determined system** $Ax = b$ **. (10 marks)**

6- i) Give an algorithm for the power method to find the dominant eigenvalue of a given matrix. **(7 marks)**

ii) Discuss the A-Stability of the trapezoidal method for solving the IVP. **(6 marks)**

iii) Give an algorithm for a shooting method to solve the BVP, **(7 marks)**

$$y'' = f(x, y, y'), \qquad a \le x \le b, \quad y(a) = \alpha \;, \; y(b) = \beta.$$

انتهـــت الأسئلة

ا.د/صلاح الجندي ا.د/عبدالحي عزوز

بسم الله الرحمن الرحيم

Assiut university	Subj. Differential Geometry	
Faculty of Sciences	14 / 9/ 2015 m	Time : 3 hours
Maths. Dept.	M. Sc. Students in Mathematics	no. (606 r)

Answer 7 (seven) questions only from the following:

1-Define the Gauss map $N : S \to S^2$ and using the differential $dN(p) : T_p(S) \to T_p(S)$ to obtain the expressions for the Gauss and mean curvatures as Det(dN(p)) and (½) tr(dN(p)), respectively.

(15 Marks)

2- State and derive Euler formula which gives the relation between the normal curvature $K_n(\vartheta)$ along a direction making an angle ϑ with a fixed direction and the principal curvatures at a point on a regular surface S. Thus, prove that the mean curvature H *at* $p \in S$ *is given by*:

$$H = \frac{1}{\pi}\int_0^{\pi} K_n(\vartheta)\, d\vartheta.$$

(15 Marks)

3- Define the gradient of a differentiable real valued function $f : S \to R$ on a regular surface S given by $X : U \subset R^2 \to S$. Then prove that

$$grad\ f = \frac{f_u G - f_v F}{EG - F^2} X_u + \frac{f_v E - f_u F}{EG - F^2} X_v$$

Thus.show that the definition agrees with the usual definition of a gradient in the plane.

(15 Marks)

4- Define the parallel surface to a regular surface X=X(u,v). Hence, find the Gauss and mean curvatures of the parallel surface interms of the curvatures K and H of a given surface.

(15 Marks)

من فضلك إقلب الصفحة Please turn over

5- Define the orthogonal families on a regular surface S. Thus, prove that the two families $\psi(u,v)=Const.\ and\ \varphi(u,v)=Const.$ are orthogonal if and only if

$$E\,\varphi_v\psi_v - F(\varphi_u\psi_v+\varphi_v\psi_u)+G\,\varphi_u\psi_u=0.$$

Henceforth, show that the two families of regular curves

$v\cos u=Const.,\ v\neq 0\ and\ (v^2+a^2)\sin^2 u=Const.,\ v\neq 0,\ u\neq \pi$

are orthogonal.

(15 Marks)

6- Define the isometry and conformal maping between surfaces.Define the tangent surface of a regular parametrized curve $\alpha=\alpha(s)$as a ruled surface.Hence show that the tangent surfaces for which the base curves $\alpha_1=\alpha_1(s)$, $\alpha_2=\alpha_2(s)$ are unit speed curves and having the same curvatures $K_1(s)=K_2(s)\neq 0$ are isometric.

(15 Marks)

7- Define the isothermal surface. Hence prove that the isothermal surface X=X(u,v) is minimal if and only if its coordinate functions $X^i=X^i(u,v),\ i=1,2,3$ are harmonic.

(15 Marks)

8- Define the normal variation of a regular surface and use it to justify the use of the word minimal in connection with surfaces with vanishing mean curvature.

(15 Marks)

9- Define the parameter of distribution of a ruled surface. Hence prove that at regular points, the Gaussian curvature of a developable surface is identically zero.

(15 Marks)

Good luck .. إنتهت الأسئلة وبالتوفيق والنجاح..د. حمدي نور الدين..

حمدي نور الدين

Assiut University
2014/2015

Final Exam of Graduate Students (Pre-Mater Degree)
Mathematics and Computer Science Students
Special Course in Computer Science M653
3 Hours – 100 Points

Faculty of Science
Mathematics Dept.

Answer the following Questions: (20 Points each)

Q1

A. Define the Bioinformatics?

B. Draw a cartoon that describes the collaboration between CS and BIO students.

C. What is the difference between **prokaryotic** and **eukaryotic** cells? Give an example of each cell.

D. What is difference between coding and non-Coding RNA?

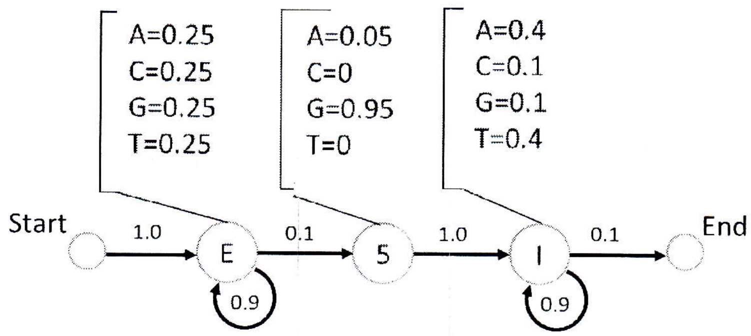
Q2

A. Describe the next figure, where **E** is exon, **I** is intron and **5** is 5'.

B. What are the main component of Hidden Markov Model?

C. Give some applications of Hidden Markov Model in bioinformatics

D. Assume you have you have sequence CTTCATGTGAAAGCAGACGTAAGTCA, Compute the score for a candidate path EEEEEEEEEEEEEEEEEE5IIIIIII



Q3

A. Illustrate how the protein sequence is converted to fixed-length vector. Why we need this mapping.

B. Assume that the candidate path is one of those three classes E, 5, I in Question Q2(D). That means, C belongs to the class E, T belongs to the class E, ..., and lastly A belongs to the class I. Give an idea to use classification to predict the path for sequence which has unknown path.

C. What is the microRNA?

D. State some dynamic programming algorithms used for bioinformatic problems.

Please see the next page

Q4.

A. Draw a cartoon describing different types of OMICS techniques.

B. State the model that is used in analyzing OMICS techniques?

C. Adjusting the P value in analyzing OMICS data is important, why?

D. Compare between microarray technique and RNA-Seq technique.

E. State the main steps in analyzing microarray data.

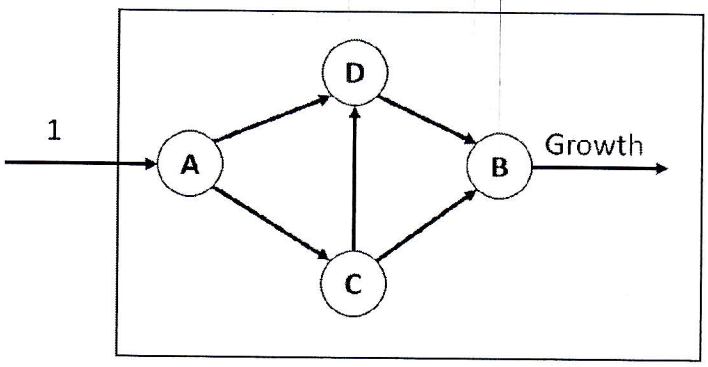
Q5

A. Apply the concept of Flux Balance Analysis for the next metabolic pathway.

B. How can you delete the reaction C → D.

C. Is this reaction is essential for the growth?

D. How can you extent this metabolic pathway to become a genome-scale metabolic model?



End of questions **Best Wishes**

Assiut University
Faculty of Science
Department of Mathematics

MSc Exam
Date: Sep 201
Time: 3 Hour

Mathematical Statistics 631M

Answer ONLY five questions from the following :

1-(a) The joint probability function of (X,Y) is given by :

$$f(x,y)=xy(x+y)/72 \quad x=1,2,3 \;,\; y=1,2 \quad \text{and } f(x,y)=0 \quad o.w$$

find the marginal distributions of X and Y and then find the expectations : $E(X+Y)$, $E(Y|X)$.

(b) -The joint probability density function of (X,Y) is given by:

$$f(x,y)=kx(x+y^2) \quad 0<x<1 \;,\; 0<y<2 \quad \text{and } f(x,y)=0 \quad o.w$$

find the value of k , the distribution function $F(x,y)$. and $E(Y|X=0.25)$.

2-(a) With reference to the random variable (X,Y) in Q 1-(a) find the distribution function $F(u,v)$ where $U=X+Y$ and $V=XY$

(b)- Given the random variables $X_1,X_2,X_3,....X_n$ with $E(X_i)<\infty$ and $V(X_i)<\infty$, if $Y=\sum_{i=1}^{n}X_i$ find $E(Y)$ and $V(Y)$ and then find $V(Y)$ when the variables $X_1,X_2,X_3,....X_n$ are independent.

3- (a) If X has the probability density function $f(x)$ and $y=k(x)$ is differentiable within the range of X for which $f(x)\neq 0$ and can be uniquely solved for x to give $x=u(y)$, prove that the probability density function of Y is given by : $g(y)=f(u(y))|u'(y)|$.

(b)- The joint probability density function of (X,Y) is given by:

$$f(x,y)=e^{-x-y} \quad x>0 \;,\; y>0 \;,\; f(x,y)=0 \quad o.w$$

find the probability density function of $U=X/(X+Y)$

4-(a) A random sample of size n from a normal population $N(\mu,\sigma^2)$, find the sampling distribution of the sample mean $\bar{x}$.

(b)- The random variables $X_1,X_2,X_3,....X_n$ are independent and having standard normal distribution, find the probability density function of $Y=\sum_{i=1}^{n}X_i^2$.

5-(a) Using the moment method to estimate the parameters of gamma distribution with probabiy density function $f(x)=x^{\alpha-1}e^{-x/\beta}/\Gamma(\alpha) \quad x>0 \,,\alpha>0,\beta>0$

(b)-Given a population having binomial distribution with parameter θ , let prior distribution of θ is beta distribution with parameters α,β find the posterior distribution of θ.

6-(a) construct the $(1-\alpha)100\%$ confidence interval for the mean value of the normal populatior with known variance .

(b)-The following data :*10,13,17,20* and *25* are random sample from a normal population , fid The *99%* confidence interval for population mean $(t_{0.05,4}=2.132)$.

أد/محمود ابراهيم محمود ، أد/ زينهم فكرى جاهين